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A

Familiar Introduction

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TO THE

THEORY and PRACTICE

OF

PERSPECTIVE.

By JOSEPH PRIESTLEY, LL.D. F.R.S.

·ΚΑΙ ΑΓΕΩΜΕΤΡΗΤΟΣ ΕΙΣΙΤΩ.

THE SECOND EDITION CORRECTED.

L O N D O N:

Printed for J. JOHNSON, No. 72, St. Paul's Church
Yard.

M. DCC. LXXX.



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TO
SIR JOSHUA REYNOLDS, KNT. F.R.S.

P R E S I D E N T

O F T H E

ROYAL ACADEMY OF PAINTING, &c.

THIS TREATISE ON PERSPECTIVE

IS, WITH GREAT RESPECT,

INSCRIBED BY

HIS MOST OBEDIENT

HUMBLE SERVANT,

LEEDS,
MARCH 20, 1770.

JOSEPH PRIESTLEY.



SIR JOHN A. REYNOLDS, BART.

PRINTED BY

OF THE

ROYAL ACADEMY OF ARTS, &c.

THIS TREATISE ON PERSPECTIVE

IS WITH GREAT PLEASURE

RECORDED BY



NUMBER 10,000

JOSEPH F. HESTER





T H E
P R E F A C E.



THE art of drawing in perspective has so many, and such obvious uses, that there can be no occasion to enumerate them. Those who want to communicate their ideas to others often feel the imperfection of *sounds*, or of characters that only represent sounds, for this purpose; and those who are desirous of receiving information find, that it is, in many cases, conveyed with unspeakably more ease and certainty, by the eye, than by the ear.

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The reason is, that every *verbal* description of any object that hath the dimensions of length, &c. must be reduced to a *lineal* one in the imagination, before any distinct idea can be conceived of it; and this is often a difficult and painful operation.

It is particularly fatiguing to the imagination to put together the parts of a complex object, when the description of each of them is made separately; for each part being conceived very imperfectly, and one part being in danger of being forgotten, while the mind is engaged in attending to another, the conception of the whole cannot but be very obscure, and inadequate. The symmetry and elegance of a pile of building is absolutely lost without a drawing, and the most masterly painters are incapable of producing any thing but the most hideous and unnatural groupes of figures, without the assistance of this art.

Of all the *imitative arts*, this of perspective is capable of being brought, and indeed has actually been brought the nearest to perfection; because it is wholly within
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the sphere of *mathematical science*, a branch of knowledge which has been most diligently cultivated of late years, and with respect to the objects of which, we are not used to acquiesce in any thing short of absolute certainty. Accordingly, we see, that there is no object of sight, be it ever so complex, which those persons who have applied themselves to the study of this art are not able to represent, just as it appears to the eye. By this means the ideas of all the beauties of nature and art are faithfully preserved, and those persons who have no opportunity of seeing the objects themselves, may study, and be delighted with them, in the works of travellers and natural historians; and philosophers can describe their machines, and their apparatus for making experiments, without any danger of embarrassing themselves, or of perplexing and confounding their readers.

As in all the other branches of mathematical knowledge, the progress of this art has been slow, but sure; and the English writers (particularly Dr. Brooke Taylor) seem to have carried it to a degree of perfection

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fection we can hardly conceive it possible to be exceeded. As much, however, is known, as we can imagine will ever be made use of.

All that is wanting seems to be a method of facilitating the attainment of this art; and in this respect I cannot help thinking there is room to improve upon all the books that I have yet seen upon this subject. Indeed the actual state of the practice of this art seems to be a proof that the attainment of it has not been sufficiently easy. For of the great numbers, both of ladies and gentlemen, who learn to draw, and even take pains to draw with elegance, there are very few who so much as attempt to learn perspective. Though, therefore, they be able to copy prints, and the designs of others, and to execute these drawings in a masterly manner, they are utterly incapable of *designing* any thing themselves, or of drawing from nature; so that a person who can even draw human figures, in every variety of attitude and passion, shall not be able to take a correct drawing of a table, a chair, or the most simple

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simple machine, or instrument, that requires nothing but straight lines.

Of those gentlemen who travel, or go long voyages, how few are furnished with the rudiments of this art, for the exercise of which they have perpetual occasion, if they would preserve the ideas of the memorable scenes and objects they meet with, or communicate any just notion of them to their friends. How many philosophers, and even persons who have been no mean proficient in other branches, both of speculative and practical mathematics, do we hear complaining of their ignorance of this art, and of the difficulties they have met with in their attempts to learn it; so that they are incapable of making a drawing of the apparatus they make use of in their experiments, and are always obliged to employ a professed artist for this purpose. These difficulties have discouraged numbers, who have not scrupled to give the very great price that books of perspective generally sell for, and who have been willing to take a good deal of pains in the study of them.

Though,

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Though, for my own part, I got a general idea of the theory of perspective pretty early, at the time that I attended to other branches of mathematical science, I was not capable of making a draught of any thing, till I was under a necessity of having original drawings of electrical machines and apparatus, and was in a situation where I could not find any person to make them for me. At first I puzzled myself with several mechanical methods of drawing; but though I made considerable improvements in some of them, I was obliged, at last, to have recourse to the rules of perspective. I found them, however, so immethodically digested, or so insufficiently explained, that, in several cases, I was able to investigate the rule myself, from considering the nature of the thing, sooner than I could find it in the books; and after all, the drawings that I did make at that time were executed when I had a very imperfect knowledge of the art.

The embarrassment I then found myself in, made me attend to the subject afterwards,

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wards, when I was more at leisure for it. Having struggled with the difficulties myself, and writing while the idea of them is fresh in my memory, I hope that I have been better able to obviate, or remove them, for the benefit of others. I have been willing, however, to make the attempt; and I flatter myself that any person, of the age and qualifications of those who ever think of learning to draw, may, by the help of this treatise, without any instructor, make themselves masters of every thing that is essential to this art. Less than a week, I am pretty clearly of opinion, would be sufficient for a master of the art to instruct another in it, in the method here laid down; and a few hours would be sufficient to give a person, who has a previous knowledge of geometry, a perfect idea of all the real varieties that can possibly occur in the practice of it.

Nor will this be any matter of wonder when it is considered, that after the preparation of the drawing-board, the whole of this art consists in drawing the perspective appearance of lines in no more than

five

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five different varieties of position, and in fixing points at given distances in those lines.

In the method described, Part IX, nothing is requisite, but to take the elevation and declination of any *single point* in an object, so that the whole mystery of it might be learned in a few minutes. I therefore cannot help wishing that mathematical instrument-makers would construct some cheap instrument for this purpose. The contrivance would be very easy, and it would greatly facilitate the practice of this useful art. Whether this method be explained in any other treatise I cannot tell. It occurred to me from considering the nature of the thing; I practised it before I was acquainted with any other; and the method, both with respect to theory and practice, is intirely independent of the common method, and abundantly more simple. The common perspective is founded on the consideration of *planes* and *lines*, but in this *points* only, in which they terminate, are regarded. But it is better adapted to the purpose of drawing large objects, in
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the open air, than small objects within doors.

The reader will observe that, besides the rules of *orthographical* perspective, laid down in Part X, this treatise contains a description of three distinct methods of drawing in common or *scenographic* perspective, each of which may be learned independent of the others.

That which is of the most general application is contained in Parts III, IV and V; and a summary view of it is given in Part VI. By this method a person will be able to draw the perspective appearance of all objects whatever, their dimensions and distances being previously known. There is no occasion for their being in view, or for their *ichnography* being taken.

The second method, contained in Part VIII, teaches how to draw the appearance of objects from their *ichnography*, or ground plan. The rules of it are exceeding few and easy, but the true dimensions of things must be taken before their *ichnography* can be drawn. If any person would chuse to confine himself to this method;

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thod; after learning to draw the appearance of objects on the ground plane, he must consult Part V. Sect. II. Case I. for a method of raising perpendicular altitudes upon any part of that plane; and then the method will be complete.

The third method is that mentioned above, which supposes the objects to be in view, and by finding the situation of points, determines the appearance of objects. This must be done by an instrument, but one single rule comprehends the whole practice.

Mathematicians will perhaps be surprised to find so little *theory* in this treatise; but of this I have made no parade, contenting myself with giving a satisfactory reason for every essential part of the practice; and if these satisfactory reasons be given, it is all that the reader can reasonably require. Dr. Brooke Taylor, and other geometricians, appear to me to have made the theory of perspective much more extensive and complex, than the *practice* requires; and to have introduced more *technical terms* than are really necessary for this purpose. Con-
sidering

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sidering perspective as a branch of geometry, I am far from blaming the conduct of those great writers ; but I would not have the young artist be discouraged, by imagining that he must necessarily make himself master of their works, in order either to practise this art, or to be fully convinced of the reasons on which it is founded.

The reader is indebted for part of this work to Mr. Joseph Priestley, of Halifax, from whose knowledge of mathematics in general, and of this branch in particular, I once expected a much more complete and elegant view of the theory of this art ; and I do not yet despair of his undertaking so useful a work. He drew up the *general view of the theory of perspective*, prefixed to the Notes, and wrote all those paragraphs in which the propositions in that piece are referred to. He gave me the method of measuring a line oblique to the ground plane, described p. 34, with the demonstration of it, and the method of drawing circles described Case I and II. Part VII. He was also so obliging as to assist me in revising and correcting the whole work.

Let

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Let it be observed, that I term this treatise only a *familiar introduction* to the theory and practice of perspective. It is therefore by no means intended to supersede other valuable works, that contain a greater variety of examples, and a detail of particular processes, which are highly useful to those who have much practice in this art. I flatter myself that by the help of this *introduction*, those books will be much better understood, and more useful than ever.

As a friend to the *arts*, but more especially to *science*, which is greatly indebted to this particular art, I shall think myself happy, if, either by what I have written myself, or by recommending the writings of others, I shall contribute to make the business of perspective more generally understood and practised. Every boy that is capable of being taught writing and accounts, might be perfectly instructed in all the rules contained in Parts III, IV, and V, of this Introduction, in a few weeks. They take up no more than 24 pages of the work, not concisely written. And
every

THE PREFACE. xv

every schoolmaster, who is as yet unacquainted with the rudiments of this useful art, might make himself master of them in a few evenings.

There is no occasion to trouble every boy with the *theory* of perspective; but I would have all young persons, without exception, made ready in the *practice*. Those who apply to any branch of the mathematics, may learn the theory afterwards. And if a person were taught no more than the *necessary rules* in his youth, he might learn *particular improvements* in drawing at his leisure, whenever he should have occasion for them.

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FAMILIAR INTRODUCTION
To the THEORY and PRACTICE of
P E R S P E C T I V E.

P A R T I.

*Of the Instruments that are of Use in the
Practice of PERSPECTIVE, and the
Application of them.*

A Person who proposes to practice the art of drawing in perspective must provide himself with a *drawing board and square*, and a *case of mathematical instruments*, consisting of a pair of compasses, a set of scales, a sector, a protractor, and a drawing pen.

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The *drawing board* is made in the form of a square, or parallelogram; and if any one of the angles be a right angle, it will be sufficient, provided the square which is used in drawing lines upon it be always applied to one of the sides that contain that right angle; and lines may be drawn in all possible directions without using any other side of the board. It will be very convenient for a person who practices drawing much, to have several drawing boards, of different sizes, for the sake of having greater variety in the compass of his draughts.

The *square* to be used along with the drawing board is a flat ruler, as *a*, Fig. 1, at one end of which are fastened two transverse pieces; one of them, *b*, fixed at right angles to it, and the other *c*, moveable, so as to be fixed at any angle required. These transverse pieces are applied close to the side of the drawing board, while the ruler lies upon it; and by sliding them along the board, lines may be drawn parallel to one another with much less trouble than by the help of a parallel ruler. By the fixed trans-



transverse, lines may be drawn parallel to one side of the board and perpendicular to the other; and by the moveable transverse, lines that have any degree of obliquity to the sides of the board may be drawn parallel to one another; and if, without moving the transverse, the ruler be removed to the other side of the board, lines may be drawn perpendicular to them. But if the obliquity be very great, it will be impossible to apply the square, so as to intersect the lines at right angles in some parts of the board. In this case, recourse must be had to a *parallel ruler*.

A great variety of *scales* is useful, in order to lay down lines of any given length, in whatever proportion is most convenient, with respect to the size of the drawing, &c. If none of the scales happen to suit the purpose, recourse must be had to the *line of lines* upon the sector: for, by the different openings of that instrument, a line of any length may be divided into as many equal parts as a person chuses.

The only lines that are necessary to be attended to upon the sector, besides these, for the purpose of drawing, are the two *lines of tangents*; one of them marked T, beginning at the center of the instrument, and ending with 45 at the extremity of the leg; the other marked *t*, beginning at the distance of about one-fourth of the radius from the center, and reaching a little beyond 75, near the extremity.

Lest a book explaining the use of the sector should not happen to be at hand, I shall just inform the person who is learning to draw, that, if he want the tangent of an angle exceeding 45 degrees, he must open the sector till the 45 on the lines marked *t* be set at the same distance, at which the two forty-fives on the lines marked T were placed; and then take the tangent required, just as he would have done, if the tangent had been less than 45, and it had not been necessary to make any other opening of the instrument.

If the sector be so constructed, as that the 45 *t* begin exactly at the distance of one-fourth

P E R S P E C T I V E. 5

fourth of the radius from the center, and consequently the tangent of 45° t , be exactly one-fourth of 45° T ; there will be no occasion for making any other opening of the sector; for the tangent of any degree exceeding 45° being taken on t , and repeated four times, will be the length of the tangent required.

Besides the small sector in one of the common pocket cases of instruments, I would advise a person who proposes to learn to draw to get another, of one foot radius, at least. Two sectors are, in many cases, exceedingly useful, if not absolutely necessary; and I would not advise a person to be sparing of expence in procuring a very good instrument, the uses of which are so various and important.

The *protractor* is generally a circle, or semi-circle, the limb of which is divided into 360 parts, called degrees. Sometimes, however, a parallelogram, about the size of a common scale, is used as a protractor. In this case the edges of it are divided in the same manner as if it had been part of a

A 3

circular

circular piece, and the lines drawn from the center to the circumference; but it is easier to lay down any angle with accuracy from a circular, or semi-circular protractor.

A *drawing pen* is necessary, because a common open pen can hardly be applied to the edge of a ruler without blotting the paper when it is taken up; besides, by the help of a drawing pen, lines may be made of precisely the same thickness throughout, and of whatever thickness a person pleases.

Black lead pencils are very useful, in order to draw lines that are of no service, but as a direction to draw other lines by them; because, when they have answered this purpose, they may be taken out with a few crumbs of soft bread.



*The Definition of necessary technical Terms,
and the Preparation of the Drawing-
Board.*

PERSPECTIVE is the art of delineating objects as they would appear upon an upright plane, interposed between them and the eye; for instance, as they appear upon a pane of glass when they are seen through a window. In this manner it is evident, that the images of objects of the same size will occupy more or less space, according as they are nearer or farther off; and the great difficulty of drawing a number of objects consists in giving them these proportions. By this art, therefore, the pictures of objects are made to exhibit the same appearance upon one plane, and at the same distance from the eye, that they have in nature upon different planes, and at different distances; which is a species of imitation that is particularly pleasing to all persons.

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To make the practice of this agreeable art the more intelligible, I shall suppose that I am about to make the perspective drawing of a number of objects, as *a, b*, Fig. 2. which require every variety in the practice; and shall minutely describe every part of the process I make use of for this purpose. [A]

The first thing I do is to fasten a sheet of paper upon my drawing-board, by bits of wafer or sealing wax, at each corner, in order to make it lie flat and steady. The surface of this paper is designed to represent the plane on which the objects are to be drawn, and which is generally called the *perspective plane*. In other words, it represents the glass window, which I before supposed to intervene between my eye and the objects, and upon which their appearance was to be drawn. This appearance I am now to copy on my paper.

In order to this, I draw, or suppose to be drawn, upon the plane on which the objects stand, called the *ground plane*, a line AB, Fig. 2, on which the perspective plane, through which I view the objects,
is

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is supposed to stand. I then take my drawing-board, and, by the fixed transverse of the square, draw the line AB, Fig. 3, to represent it. This line I, therefore, call the *ground line* of my piece.

After this I note the point C, Fig. 2, in that part of this ground line which is nearest to my eye; and upon it I raise, or suppose to be raised, the line CE, perpendicular to the other. Also, upon my paper, Fig. 3, I draw the line CE, to represent it.

The next thing I do is to measure the height of my eye above the plane on which the objects stand; and supposing it to be six feet, I set off from C to D, Fig. 3, the length of six divisions, from any scale that I think proper, and draw the line FG parallel to the ground line AB. This line, being drawn at the distance of the height of my eye, will represent a plane passing through my eye, and seen edgewise; and being parallel to the horizon (supposing that I stand upright) it is called the *horizontal line*. The point D in this line, to which my eye is directly opposite, is called
the

the *point of sight*. It is that point in the perspective plane which is the nearest to my eye.

When I have thus drawn upon my paper the horizontal line, I measure the distance at which I stand from the imaginary perspective plane; and, supposing it to be nine feet, I mark a point E, Fig. 3, in the perpendicular line, at the distance of nine divisions from D in the horizontal line. I also set off the same distance both ways from the point of sight D, along the horizontal line to F and G.

This being done, my board and paper are prepared for the delineation of the objects I propose to draw. Or, if I please, I may draw the lines HIKL, to bound the picture; but this may as well be omitted, till the drawing be compleated.

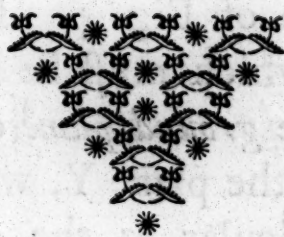
N. B. In all that follows, I shall suppose the drawing-board to be prepared in this manner; at least, that as many of these lines are drawn, and as many of the points fixed,

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fixed, as the purpose of the picture requires. I shall, therefore, hereafter omit the repetition of this part of the process. Let it be observed, also, that the letters A, B, C, D, E, F, and G, always keep the same places, and have the same uses in all the figures belonging to this work.

A general idea of the nature of perspective, and of the preparation of the drawing-board may, perhaps, be more clearly understood by means of Fig. 4. In this figure the plane X, which must be laid flat upon the table, is the *ground plane* on which the objects stand; the plane Y, which must be raised perpendicular to the other, is the *perspective plane* upon which they are to be delineated, and the part Z, which must also be raised perpendicular to the table, represents the spectator, the line *ab* being the height of his eye above the ground plane. Also *bC* is the distance at which he stands from the perspective plane Y, on which the same lines are drawn as in Fig. 3, besides others which will be explained in their proper place. If now the plane Y be

be supposed to be transparent, so that objects situated on the plane X could be seen through it, their appearance on this plane Y would be the perspective representation required.





To find the perspective situation of right lines upon the ground plane.

THE drawing board being prepared as has been already described, I proceed to draw upon it the perspective appearance of all the objects that are situated beyond the perspective plane.

Since the figures of all objects are contained under lines, bounded by points, *i. e.* under lines terminating in different places, it is evident, that the whole art of perspective consists of nothing more than a method of finding the situation of all lines upon the perspective plane, and of cutting those lines in any given proportion, according to the lengths required. Curves are no exception to this observation, since they are drawn by fixing a sufficient number of points in right lines, and joining them by a steady hand. In geometry, they are considered as consisting of an infinite number of right lines. I shall, therefore, in the

the first place, explain the method of finding the perspective situation of right lines running in all possible directions, beginning with those which are drawn upon the ground plane.

All right lines drawn on the ground plane are either parallel to the ground line, as *cd*, Fig 2, perpendicular to it, as *di*, or oblique to it, as *ml*.

SECTION I.

To find the perspective situation of right lines parallel to the ground line.

IF the lines to be drawn in perspective be parallel to the ground line, as *cd*, Fig 2, they will be parallel to it on the perspective plane, and consequently parallel to one another. [B]

Being sensible, then, that the image of the line *H'I*, upon the ground plane *X*, Fig. 4, must be drawn parallel to the ground line *A'B*, I only want to know at what distance it must be drawn from it.

To

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To ascertain this, I measure the perpendicular distance CM of the given line from the perspective plane; and finding it, for instance, to be three divisions, I set off that distance from C to *q*, and draw the line G*q*, cutting the line CD in *m*; and the line *bi* drawn through *m*; parallel to the ground line, AB is the true perspective situation of the line HI required. [C]

If I would draw another line, as OP, farther on the ground plane, and parallel to the ground line, after having drawn *bi*, representing HI; I have no occasion to measure the distance of this line from the ground line, but having taken the distance KM, of these parallel lines from one another, I set it off on the ground line from *q* to *r*; and drawing Gr, so as to cut DC in *k*, I draw *op* through *k* parallel to *bi*, and that is the line required.

If I chuse to draw a number of these parallel lines and at the same distance from one another, as in Fig. 5, I continue to set the same distance from C to *a*, from *a* to *c*, from *c* to *e*, &c. along the ground line

line AB, and drawing the lines Ga , Gc , Ge , &c. I find their intersections with the perpendicular CD at b , d , f , h , &c. then drawing lines through those points, parallel to the ground line AB, I have their perspective situations. [D]

If I chuse the distances of these lines to be unequal, I set off unequal divisions along the ground line.

In this manner may the distances be fixed, at which any lines upon the ground plane, parallel to the ground line, may be drawn; and when a method hath been explained of cutting these parallel lines, in any length required, then all such lines as cd , Fig. 2, may be determined.

SECTION II.

To draw the perspective situation of right lines perpendicular to the ground line.

IF I have occasion to find the perspective place of a line that is *perpendicular* to the ground line, as *di*, Fig. 2, I measure its distance from the perpendicular *CE*, and setting it from *C*, upon the ground line, I join that point, and the point of sight *D*, which gives the situation required.

For example; in Fig. 4, if I would draw the perspective appearance of a line, *QD*, which is perpendicular to the ground line *AB*, and parallel to *CD*; I take the distance *QC*, and set it off *C* to *q*; and then *Dq* is the line required. For the same reason, *Dr* represents the line *DR*; also *Ds* represents *DS*; and *Dt* represents *DT*, all drawn at equal distances, and parallel to one another.

Since all lines perpendicular to the ground line, when infinitely produced, seem

B

to

to meet on the perspective plane in the point of sight D, it is called the *vanishing point* of those lines. [E]

SECTION III.

To draw the perspective situation of lines oblique to the ground line.

WHEN the line, whose perspective representation I want to fix, is *oblique* to the ground line, as *ml*, Fig. 2, and CI, Fig. 6. I measure the degree of obliquity by the angle it makes with a perpendicular (as CE) to the ground line; and whatever that degree is, I make the angle at E with the perpendicular CE equal to it, and on the same side; and to the point in which the line containing the angle falls upon the horizontal line, I draw the line required; for that is the vanishing point of the given line, and of all its parallels.

Thus supposing the angle DCI, in the plane X, Fig. 6, to be 30 degrees, I make the angle DE*i*, in the plane Y, on the same side of the perpendicular, of the same quantity,

tity, and i will be the vanishing point of the line CI; *i. e.* it will be the point in which that line, infinitely produced, will seem to meet the horizon, and also that in which all its parallels will seem to meet it.

N. B. If it be more convenient (as supposing E to be without the bounds of the paper) the tangent of the angle of declination from the perpendicular may be set off along the horizontal line, from the point of sight D, the distance DE being made the radius.

In Fig. 7, a number of parallels are drawn to the same point a , 30. degrees to the right hand of the perpendicular CD; and the points in which they meet the ground line are equidistant from one another. [F]



To draw the perspective situation of lines not situated upon the ground plane.

ALL lines not situated upon the ground plane are either perpendicular, parallel, or oblique to it; and those that are parallel may be drawn by the help of those that are perpendicular to it; so that there remain only two varieties in the lines that are the subject of this chapter.

SECTION I.

To draw lines perpendicular to the ground plane.

IF the line I have to draw be perpendicular to the ground plane, as *ce*, *df*, *ih*, and *lk*, Fig. 2. and *HK* in the plane α raised upright, Fig. 6. it must be represented by a line perpendicular to the ground line, wherever it is situated. If, therefore, the

the seat of the line on the perspective plane be given, as at *b*, in the plane Y, Fig. 6, representing H in the ground plane X, I raise the perpendicular *bk*, and somewhere in that line continued will the line required terminate. [G]

SECTION II.

To draw the perspective situation of lines oblique to the ground plane.

IF a line, whose perspective situation I want, has an elevation above the ground plane, but no declination from the perpendicular; making DG equal to DF radius, I set the tangent of it from D to *a*, Fig. 9, in the perpendicular CE; or, which produces the same thing, I make the angle DG*a* equal to the elevation above the horizon (in this instance 15 degrees) and *a* will be the vanishing point of the line required, and also of all other lines parallel to it, as *ga*, *fa*, *ea*, *ba*, *ca*, *da*; all which represent lines standing upon the ground line, parallel to one another, having the same elevation above the

the horizon, and no declination from the perpendicular.

If the line, whose perspective situation is wanted, be both oblique to the ground plane, and have a declination from the perpendicular, as mk , Fig. 2, or CK , Fig. 6; I first measure the degree of declination (*e. g.* 30 degrees) and making the angle DEi equal to it, find i in the horizontal line, which is that point in which the line upon the ground plane, perpendicularly under the given line, would meet the horizon, and, through i , draw the line ig perpendicular to FG . Then taking iy equal to iE , I make the angle iyg equal to the elevation (in this case 20 degrees) which gives g for the vanishing point of the given line CK ; and, consequently (joining C and g) Cg will be the line required. [H]

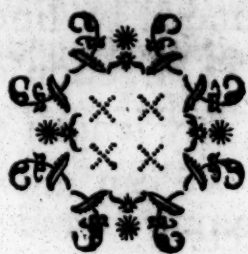
In Fig. 10, the line Ca is made to decline 25 degrees from the perpendicular CE , to the right hand; and to have a depression of 15 degrees below the ground plane of the picture; and parallels are drawn to it at the distance of one division from each other.

The

PERSPECTIVE. 23

The depression is made in the same manner as the elevation, the angle being taken on the contrary side of the horizontal line.

In Fig. 11, the line *Ca* declines from the perpendicular 20 degrees, it has an elevation of 10 degrees, and the parallels are drawn at equal distances.



PART



V.

To fix points in perspective lines, or the doctrine of perspective measures.

IN the two preceding parts, I have given directions for drawing perspective lines, and their parallels, in all possible directions. All that remains to be done, in the whole business of this art, is to *divide* these lines, so as to intercept any lengths that may be required in them. For since, as was observed before, all objects are contained in, or bounded by lines; if we can find the situation or direction of those lines, and cut them in any length or proportion that is required, we can draw the outlines of any given objects.

By the intersection of lines, *points* may be fixed in every possible situation; and *curves* are drawn by finding a sufficient number of points, and joining them with a steady hand. If, indeed, it should happen,

pen, that the perspective curve is a circle, it will be most conveniently drawn by the help of a pair of compasses.

S E C T I O N I.

To divide any line lying upon the ground plane, in any proportion required.

C A S E I.

To divide a perspective line lying parallel to the ground line.

DIVIDE the ground line, and lay a ruler from those divisions to the point of distance in the horizon. Thus, in Fig. 4, let *bi* be the perspective line given; if from any point in it, as *u*, I would cut off any particular length, *e. g.* two divisions, I first draw the line *Gu*, and continuing it to *C*, set off two measures from *C* to *s*; and then drawing *Gs*, I cut the line *ui* at *n* in the proportion required; for *un* will be two measures, corresponding to *Cs*.

In

In the same manner is ba in Fig. 12, made equal de , or three divisions, and bc to two divisions.

Lines parallel to the ground line may also be divided by marking the divisions upon the ground line, as before, and drawing lines to them from the point of sight D . Thus, if in Fig. 4, I draw Ds through u , and set off two divisions from s to t , the line Dt will go through the point n , making un equal to two divisions, as before.

N. B. In this case, and in all that follow, if from any point, as u , I want a line of any given length, and the line from which it is to be cut be not previously drawn; I must, first of all, draw, with a black lead pencil, or something equivalent to it, a *faint stroke* in the direction required; and when I have found the exact length, must take out the rest. [I]

CASE

C A S E II.

To divide a perspective line that is perpendicular to the ground line.

LET DC, Fig. 4, be the perpendicular proposed, and let it be required to cut off mC equal to two divisions. To do this, I set off Cq equal to two divisions, and draw the line Gq , cutting the line DC in m ; making mC equal to two divisions, as was required.

If, from l , in the line Dt , which is another perpendicular to the ground line, I want to cut off two divisions towards t , I draw the line Gl , and continue it till it cuts the ground line, which is here at C ; and setting two divisions from C to s , draw Gs , which cuts the line Dt in n , the point required. [K]

All the perpendiculars Dd , Dc , DC , Da and Db , in Fig. 13, are divided in the very same manner; the divisions being set off from the place where those lines meet the
ground

ground line, and other lines being drawn from them to the point of distance F or G; and this figure will show, that it is a matter of indifference which of the points F or G be made use of for this purpose. It is only more convenient to take that point of distance which is on the same side with the line to be divided. Thus, if I would divide the line Db , I set the divisions from b towards a , C and d ; and intersect it by drawing Ga , GC , Gc and Gd . But if I had wanted to divide the line Dd , I should have set off the divisions the contrary way, and have made the intersections from F.

C A S E III.

To divide a line that is oblique to the ground line.

LET Ci , in Fig. 6, be the line given, and let it be supposed to make an angle of thirty degrees with the perpendicular CD . If I want to cut off seven divisions from the point C, I set seven divisions from C to b ; then, taking the distance from the
vanishing

vanishing point i of this line, to the point of distance E in the perpendicular DE , and setting it off along the horizontal line to y , I draw the line yb , which will intersect the line Ci in b , the point required; Cb being equal to seven divisions.

If from b I have occasion to cut off any number of divisions more, I set them off from b towards A ; and, drawing lines from y to those divisions, I find the points required.

In the same manner may any other parallels to this line be intersected; and if they are to be intersected in a similar manner, the same line will serve for them all, as in Fig. 14, where the line ad , aC , and ab , are all divided by lines drawn from the point x to the divisions marked upon the ground line C , b , and e . [L]

If I want nothing but a single point in a given situation upon the ground plane, I must find its place in the perspective plane, by supposing it to be the intersection of two lines, the situation of which is given;

as by supposing one of them to be drawn through it perpendicular to the ground line, and the other to be a line drawn from the point of distance, and intersecting it at a given distance from the ground line. Thus if I want to find the perspective place of the point U , in the plane X , Fig. 4 (as if I have occasion to fix the foot of a column in that place) I measure the distance UM from the perpendicular CD ; and setting it off from C to s , I draw Ds , and conclude, that this line must go through the perspective place of the given point U ; because every point in this line Ds is at the distance of the point U from the perpendicular CE . I then measure US , the distance of the point U from the ground line, and setting it off from s to C I draw GC , which will cut the line Ds in the point u required.

Otherwise, this point may be determined by supposing one of these intersecting lines to be parallel to the ground line; for then I measure the distance US from the ground line, and, by a preceding rule, draw the line hi parallel to the ground line. Then measuring the distance UM from the
perpen-

P E R S P E C T I V E. 31

perpendicular, I set it off from C to *s*, and drawing the line D*s* to intersect the line *bi*, I find the point *u* required.

It is evident that any point upon the ground plane might, also, be found, by drawing lines making *any given angle* with the perpendicular, and intersecting them at proper distances from the ground line; but the methods here described are much easier.

SECTION II.

To divide lines not lying upon the ground plane.

CASE I.

To divide a line perpendicular to the ground plane.

IF *ac*, Fig. 15, be the image of a line perpendicular to the ground plane, and I have occasion to cut off from the point *a* in it any number of divisions; for example 5, I take at random any point *d* in the horizontal

horizontal line, and generally (as being more convenient) beyond the bounds of the picture; and from thence draw a line de to any point e , taken at pleasure in the ground line. I then set off 5 divisions from e to f , in the perpendicular ek , and draw the line fd . After this I draw a line from the foot a of my object, parallel to the ground line, cutting de in the point i ; and raising a perpendicular from i to the line df , find b ; and this line ib contains the 5 measures required; which I, therefore, set off from a to b in the given line ac . Thus ab is equal to fe .

The line de and ek , being once drawn, will serve to measure any other lines perpendicular to the ground plane, always drawing a line parallel to the ground line from the foot of the object to be measured, to the line de , and then proceeding in the method described above.

In order to measure objects with the greater exactness, the point d should be taken at a considerable distance from the
perpen-

perpendicular *ek*; but the measures will be the very same at whatever distance it be taken.

If a considerable number of lines are to be raised, all of the same height, standing in a right line, a line drawn from the vanishing point of that line, to the top of any of them, will touch the tops of them all, which fixes their height with very little trouble. For their tops, being all in a line parallel to the line that goes through their bases, a line going through them must have the same vanishing point as if it had been situated upon the ground plane.

Thus the lines *a, b, c, d, e, f, &c.* Fig. 16, represent rods of equal heights standing upon a line, the vanishing point of which is at *x*.

Also, if a number of objects be of the same height in different parts of the picture; yet, if they all stand upon a line parallel to the ground line, they must all be drawn of the same length. All those marked *d*, in this figure, are examples of it. [M]

CASE II.

To measure a line oblique to the ground plane.

LET the line be Cg , Fig. 6, and let it be supposed that there be occasion to cut off 8 divisions from the point C .

Draw CN parallel to the line yg , and considering CN as a new ground line, lay the given number of divisions, viz. 8, from C to a ; join the points a and y with a line, cutting the given one in the point k ; and Ck is the part required. Or,

From the vanishing point g of the given line Cg take gy , parallel to the ground line AB , and equal to gy ; on the ground line itself set off the given length from C to $[a]$ and a line joining $[a]$ and $[y]$ will cut off the required part Ck as before. $[N]$

The last method will be found most convenient in general, not only on account of its being more correspondent with the rules

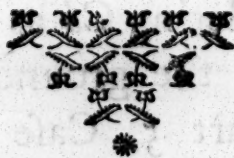
rules given in the preceding sections ; but also, that in this case, we do not encumber that part of the picture, where the images are drawn, with unnecessary lines and points.

In this example, the given line Cg stands upon the ground line AB ; but the rules above will serve for lines standing on any part of the ground plane. Should not the application immediately appear to the learner, he may take the following example :

Suppose Cg , Fig. 8, to be an indefinite line, standing on the ground plane at C , and let it be proposed to cut off a part of it, from C , equal to a given length, for instance, 10 divisions. Find the point $[y]$ by the directions given above ; and from C , parallel to the ground line, draw Cb ; then considering Cb as the image of a line parallel to the ground line, cut off from C , by Part 5, Case 1, a part Ca equal the given length, viz. 10 divisions, and a line joining $[y]$ and a will cut the given line Cg in k , the point sought.

If there be occasion to make several other divisions in the given line Cg ; in order to save the trouble of reducing those divisions to the line Cb , make Cb equal the absolute length of 10 divisions, and through k , draw bk , cutting gy produced, in d ; and d will be a measuring point for the line Cg ; so that the divisions themselves may be set upon Cb at once, and lines drawn from thence to d , will cut the given line Cg in the desired points.

N. B. The point d may be found at once, by taking gd to gy , as the height of the eye is to the perpendicular CL .





*A summary account of all the essential rules
for drawing in perspective.*

AS I have laid down the rules for drawing in perspective at full length, and have illustrated them by a considerable number of examples, it may be useful to the learner to have a *general view* of what is essential to this art, or a summary of all the rules, reduced into a small compass. This, therefore, I shall endeavour to do for him.

All lines that are parallel to the ground line, or perpendicular to the ground plane, in short, all lines that are parallel to the perspective plane, must be drawn parallel to each other. All other parallel lines meet, or have *vanishing points* in some part of that plane. If the lines lie in any direction upon the ground plane, they will vanish somewhere in the horizontal line; which is, therefore, called *the vanishing line*

of that whole plane. If the lines be perpendicular to the ground line, they vanish in the point of sight; but if they be oblique to it, or have a declination from the perpendicular, the angle of this obliquity, or declination, must be set off from the point of distance on the perpendicular, and it will find the vanishing point on the horizontal line.

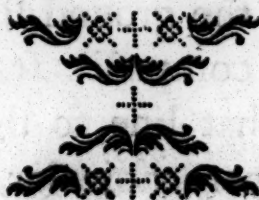
If the line to be drawn be not in the ground plane, but have an elevation above, or a depression below it, set off the angle of elevation, or depression, from the point of distance, on the horizontal line, and it will find the vanishing point on the perpendicular. If the line have both an elevation or depression, and likewise a declination from the perpendicular, set off the angle of declination as before, and through the vanishing point of declination so found, draw a line at right angles to the horizontal line; lay the extent between this vanishing point and the point of distance on the perpendicular, along the horizontal line; and from the point last found, set off the given angle
of

of elevation or depression, which will cut the line, crossing the horizontal line, at right angles, in the vanishing point required.

All the measures of lines upon the ground plane are to be laid down upon the ground line, and the *measuring point* of all lines parallel to the ground line, is either of the points of distance on the horizontal line, or the point of sight. The measuring point of any line, perpendicular to the ground line, is in the point of distance, on the horizontal line; and the measuring point of a line oblique to the ground line, is found by extending the compasses from the vanishing point of that line to the point of distance on the perpendicular, and setting it off on the horizontal line.

A line perpendicular to the ground plane is measured by a triangle, as was particularly explained, p. 31, Fig. 15. Lastly, a line oblique to the ground plane is measured by drawing new horizontal and ground lines from the vanishing point and foot of the given line respectively, and taking a measuring

asuring point in this new horizontal line; such, that its distance from the vanishing point, be to the distance of the eye from the same point, as the height of the eye is to a perpendicular let fall from the foot of the given line, to the original horizontal line; then lines drawn from this measuring point, will transfer divisions, set on the new ground line, to the given image, or the contrary.



PART



Some more particular directions, to facilitate the practice of drawing in particular cases.

SECTION I.

Of drawing circles.

I Have observed that all curve lines may be drawn by finding a number of points in their circumference, and joining them with a steady hand; and this might be deemed sufficient in a treatise, which proposes to contain nothing but the necessary rules, familiarly explained; but, since circles are lines that frequently occur in drawing, I shall give some more particular directions concerning them.

A circle is drawn with the most ease by first drawing a *square* in which it is inscribed. If the projection of the square be an oblong *trapezium*, the circle will have an oblong appearance, that is, it will be an *oval*,

oval, or more properly, as geometricians have demonstrated, an *ellipsis*. Having therefore found the image of a square circumscribing the given circle, draw therein two diagonals, and from their intersection draw lines to the two vanishing points of the sides of the square, cutting the sides of the projected square in four points, thro' which the image of the circle will pass. These will be sufficient, in general, to enable a person to draw that image with tolerable accuracy.

Let *abcd*, Fig. 17, be the image of the square, in which it is required to inscribe a circle. Having drawn the diagonals *ad* and *bc*, and found their intersection at *e*, I draw lines through *e* to *f* and *g*, the vanishing points of the sides (one of them making an angle of 40, and the other an angle of 50 degrees with the ground line) and through the points in which these lines intersect the sides I draw the curve.

If two of the sides of the square be parallel to the ground line, as in Fig. 18, and consequently have no vanishing points, I inter-

intersect the other sides, that vanish in the point of sight C, by a line parallel to the ground line, as *ef*.

If the figure be large, and very great accuracy be requisite, the images of a greater number of points in the circumference must be found; and by increasing the number of those points, the curve may be determined to the greatest nicety.

If I want to draw the appearance of a number of circles, suspended perpendicularly one above another, or different sections of an upright cylinder, I first draw the image *a* of a square with a circle inscribed, Fig. 19, lying upon the ground; then, raising the perpendiculars *e*, *f*, *g*, and *h*, I make another square *b*, at whatever height is required. This square, being raised so much above the ground plane, will appear very narrow, and consequently the circle inscribed in it will be very elliptical. If I had made another square and circle, at the height of the eye, in D, they would both have appeared as a right line.

Advan-

Advancing above the horizon, I draw the square *c*, the lines which contain it being still drawn to the point of sight *D*; but as this square and circle are above the horizontal line, it is the under-side of them that we see; and as the square and circle *d* are still higher than *c*, they are dilated still more; and the higher I carry these squares and circles, the less oval, and the more nearly circular will their appearance be.

Another example of squares and circles, in a different position from those last described, is exhibited, Fig. 20. The edges of these rest upon the ground plane, and the whole figure is oblique to the ground line, making an angle of 15 degrees with it. As the drawing of a figure like this may be thought difficult by beginners, I shall briefly describe the whole process.

Having pitched upon a point on the ground plane *d*, where I would chuse the nearest angle in the figure to stand; I draw the line *de* to a point in the horizon out of this picture, making at *E* an angle of 75 degrees (the complement of 15) with the
the

the perpendicular DE . In order to cut this line at right angles by another line upon the ground plane, I find the point f , by making the angle DEf equal 15 degrees, the complement of 75 to 90. In order to cut as much of the line fd as I want for the side of the square (suppose 4 measures) I take fb equal to fE and thereby find b the measuring point of the line fd , and of all its parallels, several of which will be wanted in this figure. Setting off 4 divisions upon the ground line, from x (the place where a line drawn through b and d would touch the ground line) I make dg equal to 4 measures. Then raising perpendiculars from d , and g , I make the line db equal 4 divisions; and, drawing the line fb , the first square is compleated. This done, I draw lines from all the angles of it ij , bk , gl , to the same vanishing point with de ; for these lines will determine the size of all the squares; and now I have nothing to do but to set off on the ground line, from y , the place where a line drawn through a and d would touch it, the distances at which I would place the squares from one another (suppose 4 divisions.)

These

These I mark upon the line *de*, from the measuring point *a*; and when I have drawn the lines *mq*, *nr*, &c. to the vanishing point *f*, and have raised perpendiculars from the points *m*, *q*, *n*, *r*, &c. to the lines *bk*, and *ij*, the squares are completed; after which the circles will be inscribed in them with ease.

All this may be done in less time than the description of it can be transcribed; and with as little trouble may the same number of circles, in any other direction whatever, be drawn.

Cylinders, being figures terminated at both extremities by circles, they may be drawn by first making a parallelopiped of the same length, and inscribing circles in the squares of each end of it; after which, the line which belongs to the parallelopiped, being drawn with a black lead pencil only, may be wiped out.

If the image of a diameter of any circle, situated any where on the ground plane,
be

PERSPECTIVE. 47

be given, the image of that circle may be determined in the following methods.

CASE I.

When the given image is parallel to the ground line.

LET ab , Fig. 21, be the given image; bisect it in s , and through s , and the point of sight C , and points of distance D , G , draw $1sC$, tsG , $2sD$; from either point of distance, D , draw lines, Db , Da , cutting $1sC$, in 1 and 4, and 14 is a diameter perpendicular to the ground line. Lay down the distances GE , DE on the horizontal line to F and H , by which, through a and b , draw $Fa1$, $Fb3$ and $Hb2$, $Ha5$ cutting tsG and $2sF$, in t , 3, and 2, 5 respectively; connect the points 1, 2, b , 3, 4, 5, a , t , with a curve, which will be the image required. [O]

CASE

C A S E II.

When the given line is oblique to the ground line.

FIND the image of a diameter which is parallel to the ground line, and then proceed as in Case I.

Thus, let $[ab]$ be the given image; continue it to the horizontal line at $[g]$; take the distance $[gE,]$ and lay it on the horizontal line to $[d]$ the measuring point of $[ab]$; from $[d]$ thro' $[a]$ and $[b]$ draw lines cutting the ground line in $[e, t]$; bisect $[et]$ in $[l]$ and join $[ld]$ cutting $[ab]$ in $[h]$ the image of the center of the circle; then through $[h]$ draw $[os]$ parallel to the ground line, terminating in $[de]$ $[dt]$ and $[os]$ is the image of a diameter of the same circle, parallel to the ground line.

By these rules, the image of a circle, lying in any known plane, may be described from the image of a diameter being given. An example of one, in a plane perpendicular

P E R S P E C T I V E. 49

cular to the ground plane, is given in the figure; where hgm is the vanishing line of the plane and cd the given diameter, thus making $gb, gr = gC$, and $hm, rk = bC, rC$; then through e , the middle of cd , draw $gef, rneq$ and $hpet$; draw bc, bd cutting gf in o and f ; draw kc, kd cutting qnr in n and q ; lastly draw mc, md cutting tpb in p and t ; and the points c, t, f, q, d, p, o, n , connected, will give the required image.

The methods delivered above become very useful when the images of a great number of parallel circles are to be drawn; for the vanishing points C, D, G, F, H , serve for drawing the representation of circles in all planes parallel to the ground plane; and g, k, r, b, m , for all those lying in planes parallel to the plane $cfdo$.

D SECTION

SECTION II.

Of different ground planes.

HITHERTO all the objects to be drawn were supposed to stand upon the same ground plane, whose vanishing line is the horizon; but in drawing landscapes, and other things, it sometimes happens, that there are other uniform planes, of considerable extent, on which objects are to be drawn. In this case, it will be convenient to draw the vanishing lines of those planes, and make use of them as horizontal lines, for the objects that are to be described upon them.

Thus, in Fig. 22, the plane *a, a, a, a*, is the common ground plane, the vanishing line of which is the horizontal line, passing through D; and to this point all lines upon the ground plane, perpendicular to the ground line, are to be drawn. But the plane *b*, has a depression of 15 degrees below the horizon. Its vanishing line, therefore, passes through H, DFH, being
15 degrees,

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15 degrees, and the plane *ccc* has an elevation of 15 degrees above the horizon. Its vanishing line, therefore, passes through *I*, the angle *DFI*, being 15 degrees.

If the plane have a declination with respect to the perpendicular, or be obliquely situated, with respect to the ground plane, its vanishing line may be found by finding the vanishing points of two lines, drawn on that plane in any direction, provided they are not parallel to one another; for the vanishing line required, will pass through both those vanishing points.





*A method of drawing in perspective from the
ichnography of objects.*

IF any person will take the pains to draw the *ichnography*, or ground plan of what he proposes to draw, expressing the true proportions of all the objects he introduces into it, it will be easy, afterwards, to reduce it into true perspective. This method I shall illustrate by an example, which will render it very easy.

Let the object to be drawn be the triangle *abc*, Fig. 23; and let its situation, with respect to the ground line, be the same that it has in this figure, being only placed on the opposite side of it, and therefore in an *inverted position*.

To find the vanishing point of any of the sides of this triangle, as [*ac*] I produce that line, till it meet the ground line in *d*, and draw *Ef*, parallel to *ad* by which means I find *f* the vanishing point required;
fo

so that, drawing fd , I conclude that the perspective representation of the line $[ac]$ must be a part of it. In order to determine in what part of this line either of the extremities, as $[c]$ is to be fixed, I lay a ruler from $[c]$ to E , which cuts the line fd in c the point required. In like manner, the perspective place of $[a]$ will be found to be in a .

By the same process, G will be found to be the vanishing point of the side $[bc]$ and b the perspective place of $[b]$. These three points being joined, the triangle is completed. But there is no occasion to find any thing more than the directions of the lines fd , and eG , together with the points b and c , for the intersections of the lines will find the point a .

The vanishing point of the line $[bc]$ might have been found by the same method, but it was unnecessary; because it was determined, by joining the points bc , which were found as the extremities of the other lines.

If the body to be drawn be a solid, I first lay down the ground plan, and when I have reduced that into perspective, I raise either perpendiculars, or lines oblique to the ground plane, from any point in the ichnography, according to the rules laid down before.

If I want no more than to fix the perspective place of a single *point* upon the ground plane, as for instance, that of [c] I draw a line through it in whatever direction I please, to the ground line, as to *d*. Then finding, as before, *f* the vanishing point of that line, I lay a ruler from [c] to the point of distance E, which cuts that line in *c*, the point required.

But a more ready way of determining the place of a single point as [a] Fig. 24, is to let fall a perpendicular *ab*, to draw the line from *b* to D, the point of sight (or the vanishing point of the line *ab*, which is perpendicular to the ground line) and then, setting off, on the ground line *bd* equal to *ba*, to draw a line from *d* to F, the

the point of distance, cutting the other line in *a*, which is the perspective point required.

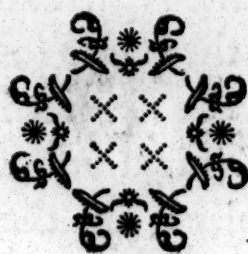
The line *ba* may be transferred to the ground line to *c*, as well as to *d*; and from thence a line may be drawn to *G*, the other point of distance on the horizontal line. This figure sufficiently shows, that both the lines *dF* and *cG* will cross the line *bD* in the very same place *a*. [P]

When objects are very complex, consisting of a great number of sides and angles, it may be the most convenient to divide the ichnography of them into small squares; and having thrown the squares into perspective, to note the termination of every line, by marking its place in the corresponding square. An example, in Fig. 25 (taken from Mr. Emerson) will make this method perfectly intelligible.

In the lower part of this figure, the ichnography of a piece of fortification is divided by squares; and in the upper part
of

of it, those squares are thrown into perspective, and exhibit a view of the same piece of fortification ; each point being placed in its corresponding square.

This is a very common method of drawing in perspective, and is called the method of *Reticulation*.





An easy method of drawing in perspective without previous mensuration, by means of an instrument to take the angles under which objects appear.

THE rules before laid down for drawing in perspective, suppose a previous knowledge of the real distances, magnitudes, and relative positions of all the objects that are introduced into the picture. But, in many cases, a person may be so situated, that this knowledge cannot be obtained; and in many cases, likewise, the labour which this method requires would be exceedingly troublesome and discouraging. I shall, therefore, in this part, describe a method of drawing any objects in true perspective, without moving from the place in which they are viewed. It is also a method of drawing that is very simple; it requires the knowledge of very few technical terms; and the rationale of it is extremely obvious.

To

To supply the place of actual mensuration, I provide myself with an *azimuth quadrant*, a *Sisson's theodolite*, or any instrument by which I can find the elevation of an object, and likewise its angle of declination from the perpendicular going through the point of sight.

Having this instrument, and placing myself at what distance I think most convenient, from any objects that I propose to draw; I lay down, upon the paper of my drawing-board, only two lines, crossing one another at right angles; one of them FG, Fig. 26, to represent the horizon, and the other CE the perpendicular, passing through the point of sight D. I also chuse whatever distance I think proper to work at, and set it off from D to E.

Having thus prepared every thing for the operation, I pitch upon any point in an object before me, as that which corresponds to α , and, by the help of my instrument, first of all, find its declination from the perpendicular to the right or left hand. Supposing it to be 10 degrees to the

the right, I set off the angle DEa equal to 10 degrees, and conclude, that the point I want to fix must be somewhere in the perpendicular ae . To find in what part of this line the point is, I, in the next place, take its elevation above the horizon, and, supposing it to be 20 degrees, I make ab equal to aE , and the angle abx equal to 20 degrees, which finds x the point required.

In this method may the situation of any other point be found, and these points, being joined by lines, the whole object will be delineated.

But if the objects to be represented contain many *right lines*, that are parallel to one another, such as occur in buildings, machines, &c. there will be no occasion to take many points; because the situations of the lines may be determined by *vanishing points*, found by the help of a few of them. Thus having found y in the same manner as I found x , and knowing that the line xy is parallel to the horizon; I produce it till it touches the horizontal
line

line in c ; which is, therefore, the vanishing point for that line, and all that are parallel to it, several of which are represented in this drawing. The distance at which these parallels are drawn, may either be guessed by the eye, or be determined with more accuracy by finding a point at one of their extremities, in the manner just now described.

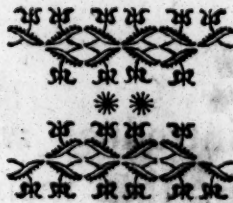
Also, if I know the angle that any line, as yz , makes with another line, as xy , the situation of which is known, I have no occasion to take any point, in order to determine the direction of it. In this Fig. xyz represents a right angle, which is that which most frequently occurs in buildings, machines, &c. To determine, therefore, the situation of the line yz , I consider that c , the vanishing point of yx , makes the angle DEc equal to 20 degrees, the complement of which, from 90, is 70. I, therefore, make the angle DEH equal to 70 degrees, and a line EH produced till it meets the horizontal line (in a place without the bounds of this picture) gives the
the

the vanishing point of yz , and of every other line at right angles with the line xy . To this point, therefore, I draw the line yz , and all the others in that figure that are parallel to it.

The point x is to the right hand of the perpendicular, and it has an elevation above the horizon; but it can require no particular instruction, or example, to be able, from this, to fix any point to the left hand of the perpendicular, and one that has a depression below the horizon.
[Q]

If I would introduce *measures* into a drawing made in this manner, or find a *scale* for the picture; I measure some one line in the original, as kf . In order to this, from c , the vanishing point of the line kf , I take cd equal to cE , which gives d the measuring point of the line; and from this point I draw two lines, one from each extremity of the line kf to ib in the line that bounds the picture, or any other line drawn parallel to the horizontal line. Then
I divide

I divide the line *ib* in the same proportion as *kf*; and by this means I get a scale, by which I can measure any other line in the picture, or insert in it other objects of any given magnitudes, according to the rules laid down above.





P A R T X.

Of orthographical perspective.

I N the methods of perspective, the rules of which have been laid down in the preceding parts of this treatise, the eye of the spectator was supposed to be placed at a definite distance from the perspective plane; they may, therefore, be called *stereographical methods*, in allusion to the stereographic projection of the sphere, in which the eye has a similar situation; or rather, since the situation of the eye, in the common method of perspective, is variable, the term *scenographic* may be more proper. But there is another method of perspective, in which the eye is supposed to be placed at an infinite distance from the perspective plane. It may, therefore, be called *orthographic*, in allusion to the method of projecting the sphere orthographically.

This method of perspective is peculiarly adapted to the description of machines, and other things, the magnitudes of which bear but

but a small proportion to the distance which they are generally viewed.

A clearer idea may, perhaps, be conceived of this method of perspective, by considering what kind of a shadow of an object would be projected upon a plane, by the rays of the sun falling perpendicularly upon it. For the distance of the sun is so great, that all its rays may be considered as coming parallel to one another.

Since there can be no more than two positions of a right line with respect to the rays of the sun, viz. *perpendicular*, or *oblique*, the perspective of lines in this orthographic method is comprehended in two cases, and that of angles in another. The position of a line, parallel to the rays, does not deserve to be called a *case*, because the image of it is nothing more than a point.

If the line be *perpendicular* to the sun's rays, *i. e.* to the visual ray, the image of it upon the perspective plane will be a line, of exactly the same length. Thus if AB, Fig. 27. be a section of the perspective
plane,

P E R S P E C T I V E. 65

plane, and I would lay down the image of any line, as *ef*, I make *cd* of exactly the same length; for it is plain that it would be the shadow of the line *ef*; the rays coming in the direction of *ed* and *fe*.

If the line be *oblique* to the visual ray, as *ce*, I lay it down with the same degree of obliquity to the section of the perspective plane. Supposing it to be 25 degrees, I make the angle *dce* equal to 25 degrees, and the perpendicular let fall from *e*, the remote extremity of this line will intercept *cd*, the image required: For it is evident that *cd* would be the shadow of the line *ce* in that situation. In other words, the image of any line is always the co-sine of the angle of its elevation above the perspective plane, the line itself being considered as radius.

I shall illustrate this method of drawing in perspective by examples, in which both the methods may be compared.

Fig. 28 represents a front view of a cube, drawn in the common method of perspective. It stands upon the ground

E
plane;

plane, and its upper side is terminated by lines drawn to a vanishing point in the horizontal line. Only two of the sides are visible in this position. The image of a circle, inscribed on one of its sides is a circle, that of the other an ellipse.

In the orthographic method, the visual rays being equally perpendicular to every part of the side that is opposed to it, all that can appear is a plain square, with a circle inscribed, as in Fig. 29.

Fig. 30 represents the same cube in the common method of perspective, in a situation oblique to the eye, one of the sides being placed at an angle of 25 degrees, and consequently the other at an angle of 65 degrees. In this position three of the sides appear, and the images of all the circles are ellipses.

Fig. 31 represents the cube, in the same situation, viewed orthographically. The side *cd* was found by drawing *ce*, Fig. 27, equal to the true length of the side, making the angle *dce* equal 25 degrees (being the
the

the angle that this side was supposed to make with the visual rays) and letting fall the perpendicular ed ; so as to intercept the line cd , the projected length required. The length of the line bc , Fig. 31, was found by taking the line de , Fig. 27, the sine of the angle of elevation of the former line. It might have been found by placing the line ce at an angle of 65 degrees, and taking the co-sine of the elevation, as before. For, reversing the figure, the line ce will be seen to be in that position, and cf equal to de will be the line required.

Bisecting the sides of the two parallelograms made upon these lines, Fig. 31, points are found for the termination of the axes of the ellipses, into which the inscribed circles are projected.

Only two of the sides of the cube are visible in this position, because one line in the figure, viz. cb , is perpendicular to the visual ray.

If the position be changed, and dx be considered as the base, it will represent the

cube resting on that single line; the line *cb* being drawn forwards; so that *cd*, or *bx* will make an angle of 25 degrees with the perpendicular; the consequence of which will be, that the upper side of the cube *cbyb* will come into view.

If the figure be supposed to rest on the line *yb*, it will represent the cube leaning still more forwards, and more of the upper side *bcxd* in view.

Instead of the cube being supposed to rest on a single line, in these cases, we may conceive it to be viewed obliquely, the eye being raised above the plane on which it stands; so that the upper side of the cube may be seen, without its being raised towards the eye.

If I would represent three sides of this cube as seen orthographically, it is evident that it must both stand obliquely, as in Fig. 31; and also that the eye must be elevated above the plane on which it stands. Suppose then, every thing else in this figure remaining the same, the eye be raised 10 degrees,

degrees, as in Fig. 32, it will be evident, that the lines cb and dx must be made shorter than the corresponding lines in the preceding figure, for being inclined 10 degrees to the visual ray, they must be lessened in the proportion of the co-sine of that angle to radius, and reduced from the length ci , Fig. 27, to cj . Also every line in the figure, parallel to them, must be made of the same length.

The lines cd and cb must now no longer be one line, for the upper side of the cube will be seen, and consequently the base would be seen too, were the figure transparent.

In order to determine the reduced lengths of the sides cd and cb , and at the same time the angle they make with each other, in this situation, I lay down the sides and angles DCb , Fig. 33, in their real size and position, and draw Ce , from the point C , in the situation in which a plane would pass through the eye and that point, in the position given before, *i. e.* so as to make the angle eCb equal 25 degrees, and eCD equal 65. Perpendicular to Ce , and through

through the point b I draw the line be , and produce it till it meet the line CD , continued, in f . Upon the center e , with the radius eC , I describe the arc Ch , and from b I set off bg equal to the angle of the elevation of the eye. Then parallel to eb , I draw gc , cutting the perpendicular Ce in c , the point into which C will be projected. Joining cb , I have the image of the side Cb , and drawing Dd parallel to Ce , to cut fc in d , I get dc for the other side. Also the angle dcb , is the projected angle required. [R]

Accordingly, in Fig. 32, I make dcb the same angle as that in Fig. 33, contained under the same sides; and completing all the parallelograms, the projection of the whole figure is determined.

If the line cd be not drawn parallel to the ground line, the angle it makes with it, added to the angle ncb , must always make 10 degrees, or any other angle that is required.

Instead of supposing the eye to be raised above the plane of the base, the image would

would be the same, if the cube were supposed to rest upon the corner *c*, the line *cb* leaning forwards in an angle of 10 degrees.

It may be convenient to apply this method of orthographic perspective in drawing very *complex figures*, when it would require a good deal of time to find the vanishing points of all the sides. If a view in common perspective be necessary, it will be easy to reduce this orthographic projection to it by the rules laid down, in Part 8, to draw the perspective appearance of objects from their *ichnography*, which is given by this orthographic method.

In drawing machines in general, there is no occasion to lay down angles and sides with this exactness. It will be sufficient to lay down the ichnography of the object to be described; as for example, the parallelogram, in Fig. 34, in whatever position may be thought most convenient, and drawing the elevation in perpendiculars over the base. Thus in Fig. 35, a hollow paralleliped, or trough, is represented
standing

standing upon the base of Fig. 34. A figure like this is perfectly intelligible, and much more easy for a workman to copy after, than any other drawing whatever ; because all the dimensions are taken from a common scale, and the imagination may be sufficiently assisted to conceive of it by good shading. It might not, perhaps, be amiss for persons who are learning to draw in perspective, to begin with this simple method, and accustom themselves to it for some time, before they attempt any other.



P A R T XI.

Of shadows.

WHEN objects are seen in a strong light issuing from one place, they project *shadows*, which it is very convenient, at least ornamental, to draw in the perspective views of them; I propose, therefore, in this part, to lay down rules for delineating these shadows, beginning with those which are projected on the ground plane on which the objects stand, as being the most common, and the most useful case.

SECTION I.

To draw the shadows of bodies on the same planes on which they are situated.

SINCE rays of light always issue in straight lines, it is evident that the shadow of any object will cover just as much space, as it would hide from an eye situated in the

the place of the luminous body. For the same reason, the shadow of any point of an object must be somewhere in a right line drawn on the ground plane, from a point perpendicularly under the luminous body, through the point that is perpendicularly under that whose shadow is required; and the precise point of this line on which the shadow will fall, will be determined by a right line drawn from the luminous body through the point of the object.

This observation supplies a rule for projecting the shadows of all bodies whatever. Find the point upon the ground plane that is perpendicularly under the luminous body. From this draw right lines through perpendiculares let fall from those points, the shadows of which are required, and intersect them by right lines drawn from the luminous body through those points.

Thus, in Fig. 36, if the luminous body be at *a*, perpendicularly over *b*, upon the ground plane, and it be required to find where the shadow of *c*, the top of the line *cd* (in one angle of the solid [x]) will fall;
since

since cd is a line perpendicular to the horizon, I draw a right line through b and d ; and drawing another right line through a and c , intersect it in e ; which will, therefore, be the place of the shadow required. Finding, in the same manner, f the place of the shadow of g , and h the place of the shadow of i , and joining the points d , e , f and h , I have the out-lines of the shadow of the whole solid [x].

If I want the shadow of the rod [y] which stands oblique to the ground plane, I let fall a perpendicular kl from the top of it, k , to the ground plane, and drawing the lines ak and bl , I find that they intersect each other at m ; which, therefore, marks the place of the shadow of k ; and since the rod is a straight one, and consequently all the points of the shadow must fall on the same right line, I draw a line from d , the foot of the rod, to m , and thus get the shadow of the whole rod.

In the same manner may be drawn the shadows of solid bodies, the bases of which project beyond the perpendiculars let fall
from

from them. First find the place where the perpendicular would fall upon the ground plane within the solid, and proceed with it, as if nothing solid had been in the case.

If the shadows of objects be made by the light of the sun, the situation of that luminary, with respect to the picture, must be determined, and this is done by the help of the following considerations.

By reason of the immense distance of the sun, it must always be supposed to be in, or over some point in the horizontal line, in which the ground plane, when extended to an infinite distance, is conceived to vanish; and the particular point in the horizontal line must be determined from the number of degrees that the sun is situated to the right or left hand of the perpendicular that goes through the point of sight; in the same manner as the vanishing point of a right line upon the ground plane, that is oblique to the ground line, is found.

If,

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If, for instance, the sun be 30 degrees to the left hand of it, I make the angle DEH, Fig. 37, equal to 30 degrees, and H will be that point in the horizontal line over which the sun may be supposed to be perpendicular; and nothing is now wanting to fix its precise place above the horizon, but to know its *altitude*, which is laid down in the same manner as the vanishing point of a line that is elevated above the ground plane. If, for instance, the sun be 35 degrees high, I make HI equal to HE; and, raising the perpendicular HK, make the angle HIK equal to 35 degrees, and K will be the sun's place.

If, now, I would draw the shadow of any object made by the sun in this situation, I suppose K to be the place of the candle *a*, in Fig. 34, and H to be *b*, the point upon the ground plane that is perpendicularly under it, and I proceed exactly as in that example. Thus, to draw the shadow of the solid [x] Fig. 36, the place of the sun being at K, I draw Ha, and intersecting it by Kb in *c*, I have the place of the shadow of *b*; and joining *ac*, I have the shadow

dow of the perpendicular line *ab*. In like manner I find *d*, the shadow of *f*, at the top of the line *ef*; and also *g*, the shadow of the vertex of the line *bi*. Then, joining these points, I have the shadow of the whole object [x]: for it is evident, from the position of it, that the shadow of any other line in the figure must fall within that of these.

When, as in this example, the place of the sun is fixed above the horizontal line, it is evident, that the shadows will be projected as falling towards the spectator, and that they will always be larger than the objects. To draw the shadows which are made by the sun, having the same degree of elevation *behind* the spectator, which will make the shadows fall nearer the horizontal line, and less than the objects; the perpendicular from H, Fig. 37, must be let fall to L, and the angle HIL must be the same as HIK in the former case; and then, if the point L be made use of, instead of K; as these points are situated on the contrary sides of the horizontal line, the shadows of objects will be represented as they would be made by the sun placed
on

on the back of the spectator; and this position of shadows is generally thought more agreeable in a picture than the other.

To find, in this manner, the shadow of the line ab , in the solid [x] Fig. 39, I draw Ha to the foot of it, and Lb to the top, intersecting each other in c , the place of the shadow. Finding, in the same manner, where the shadow of the line ef , behind the body, would fall, I join that point and c , by a line which cuts the side of the figure in d , where only the shadow begins to be visible. Thus joining a , c , and d , the shadow of the solid [x] is completed.

In both these cases, the sun is supposed to be situated either *before*, or *behind* the picture. If it be *in the plane* of the picture, it is evident that its place must be in the horizontal line, at an infinite distance from the point of sight. Consequently all lines proceeding from it may be supposed parallel to one another. To draw the shadows of objects made by the sun in this situation, draw lines parallel to the horizontal line from the foot of the object,
and

and intersect them by lines making angles with the horizontal line equal to the sun's altitude.

This practice is very easy by means of the drawing board and square. Thus if I would draw the shadow of a solid [x] Fig. 40; with the ruler of the square parallel to the ground line AB, I draw ac ; and fixing the moveable transverse of the ruler in the angle of the sun's elevation, I draw the line bc , which gives c for the place of the shadow for b . In the same manner also, I find d the place of the shadow of e ; and joining the points dc , I get the shadow of the whole figure; though it is so situated, that part of it is hid by the object. If there be ever so many lines, the shadows of which are to be found, once fixing of the ruler serves for them all. I have nothing to do but to slide it along the drawing-board, till I come to the points from which the lines are to be drawn, both for the lines that are parallel to the horizon, and those that intersect them.

To

To draw the shadow of the solid [y] Fig. 40, which lies oblique, both to the horizon, and to the ground plane, I draw ac from the foot of the line ab , parallel to the horizon; and fixing the moveable transverse of the ruler, so as to make an angle equal to the sun's altitude, I draw bc . In the same manner I also find g , the shadow of f ; then drawing gc , and joining cd , the place where the nearest extremity of the solid touches the ground, I am able to compleat the outline of the whole shadow; for the shadows of all the other lines must fall within these.

Let it be observed, that it makes no difference in the shadow, whatever be the height, or shape of objects, provided their tops, and every part of them, be in the same right lines proceeding from the luminous body. Thus the shadows of the several objects a , b , c , and d , Fig. 41, perfectly coincide, from the plane where each of them begins.

SECTION II.

Of shadows intercepted by other objects.

WHEN the shadow of a line falls upon any object, it must necessarily take the form of that object. If it fall upon another plane, it will be a right line, and if upon a globe, or cylinder, it will be circular.

If the body intercepting it be a plane, whatever be the situation of it, the shadow falling upon it might be found by producing that plane till it intercepted the perpendicular let fall upon it from the luminous body; for then a line drawn from that point would determine the shadow, just as if no other plane had been concerned.

But the appearance of all these shadows may be drawn with less trouble, by first drawing it through these interposed objects, as if they had not been in the way, and then making the shadow to ascend perpendicularly.

dicularly up every perpendicular plane, and obliquely on those that are situated obliquely, in the manner that I shall now describe.

Finding that the shadow of the upright pole *a*, Fig. 42, would have reached, in a straight line, to *b*, if it had not been intercepted by the object [*x*]; I first draw the shadow through that object, and finding that it first touches it at *c*, where it must ascend perpendicularly to the ground line, I raise the perpendicular *cd*; and since the upper surface of this body is a plane, parallel to the ground plane, I make that part of the shadow *de* parallel to the rest of the shadow upon the ground plane.

If the first side of the interposing object that the shadow falls upon be hid from the eye, as in Fig. 43, I still draw the line *ab*, as if it were visible, and where the shadow meets with it, raise the perpendicular *bg*, as before. Or, I may begin at the termination of the shadow, and tracing it back to the place *c*, where it cuts the interposing body, find the point *f*, where it left that

new surface, and draw the part fg of the shadow parallel to the rest of it, as before.

If the surface of the body interposed be an inclined plane, I find, by perpendiculars, as a and b , Fig. 44, both where the shadow enters the oblique surface, at e , and where it leaves it at d ; and then joining the points c and d , I have the direction of the shadow on that oblique plane.

If these planes change ever so often, I have only to draw perpendiculars from them all, and the direction of the shadow will be found with great ease.

If I want a shadow of only part of an object, or an object not standing upon the ground, as for instance, the part $[ab]$ Fig. 44, I suppose it to be continued to the ground, and I find where the shadow of $[b]$ falls, as at e , in the same manner as if the pillar had been no higher; and then the part at f , lying upon the ground plane, together with the part de , upon the inclined plane, will be all the shadow of $[ab]$ required.

If

If the pillar reach no higher than [b] and it had been required to find on what part of the inclined plane its shadow would terminate; I suppose the object to have been continued higher, till some part of the shadow had fallen upon the ground plane, as at *f*; then drawing the perpendicular *bd*, I find *d*, the place where the shadow would have left the inclined plane, if the object had been continued; and the point *e*, where a line from [b] cuts the line *dc*, is the termination of the shadow required.

If the object upon which the shadow is thrown be circular, the shadow will be circular. It must, therefore, be drawn with just such a degree of curvature, as the body itself would have had, if it had been intersected by a plane in the direction of the shadow. Thus, in Fig. 45, when the shadow of the pole comes to the cylinder, lying obliquely to the horizon, it goes over it, in the form of an ellipse, such as would have been made, if the cylinder had been intersected by a plane in the direction of the shadow. The curvature may be

drawn with sufficient accuracy, by observing where the shadow cuts the line representing the base of the cylinder, and raising a perpendicular on that point, equal to the cylinder's apparent diameter, for this will find the place where the shadow goes over it. Thus, the shadow crossing the base of the cylinder at *a*, I raise the perpendicular *ac*, cutting the line representing the highest part of the cylinder in *c*, and the shadow will pass over that point.

SECTION III.

Of faint shadows.

WHEN objects are not supposed to be viewed by the light of the sun, or of a candle, but only in the light of a cloudy day, or in a room into which the sun does not shine, there is no sensible shadow of the upper part of the object, and the lower part only makes the neighbouring parts of the ground on which it stands a little darker than the rest. This imperfect, obscure kind of shadow is easily made, being nothing more than a shade on the ground,

ground, opposite to the side on which the light is supposed to come; and it may be continued to a greater or less distance, according to the supposed brightness of the light by which it is made. It is in this manner (in order to save trouble, and sometimes to prevent confusion) that the shadows in the greater part of drawings are made. Examples of these shadows may be seen in Fig. 15, 16, 26, and some others in this work.

SECTION IV.

To draw the reflected images of objects in water.

THE appearance of the image of any object, as viewed in a plain mirror, is exactly of the same dimensions with the image itself, but inverted with respect to it. If, therefore, from the foot of the representation of any object, standing near the water side, a similar image be drawn downwards, in an inverted position, of the same length and form with the representation itself, the part that falls upon the water is
the

the reflected image, or as much of it as can be seen in that situation.

Thus *b*, Fig. 46, is the reflected image of the object *a*, standing close to the water side; *c* is as much of the image of *d*, standing upon the plane, at some distance from the water, as can be seen in that position; and *e* is as much as can be seen of the image of *f*, standing upon an eminence near the water.

If the object be oblique to the ground plane, as *h*, give the image *g* an equal degree of obliquity, and that will exhibit the true image required.

These objects are uniform throughout. When objects of other forms are to be represented, let it be remembered, that, since these images must be *inverted*, that part which is highest in the object must be lowest in the image, and *vice versa*. Thus the tops of trees, and the heads of men must be drawn the farthest from the brink of the water.

PART



P A R T XII.

General advices and directions, relating to the art of drawing in perspective.

IT discourages many persons, who would be glad to learn the art of perspective, to think that they must be obliged to measure every thing that they draw ; and, it is true, that all the rules of perspective, except those laid down in Part ix. do suppose the objects to have been measured, and no drawing can be perfectly accurate without it. But where extreme accuracy is not required (as indeed it very seldom is) there is little occasion to measure any thing ; and yet a person who has a just idea of the nature of perspective, will make a drawing infinitely more just, and agreeable, than another person can, who shall even measure every thing.

This is more especially the case where a great number of parallel lines, cutting each other at right angles, or any given angles, are to be represented ; such as always occur

cur in drawing buildings, machines, furniture of houses, &c. For since all parallel lines, not parallel to the picture, have the same vanishing points, if a person only know where to fix those points, so that lines drawn from them shall make any given angle with one another, his drawing, by the help of those points, will be far more agreeable to truth, and look infinitely better, than if he had gone to work without that previous knowledge to guide him. Besides a perspective drawing is always made in far less time than any other.

A person who understands this art, has nothing to do but to judge by his eye of the proportion of objects, and all his lines are sure to be perfectly true upon that supposition; and, therefore, the whole will be consistent with itself. But the random designer may, in one draught, make a hundred different suppositions; and as it is a great chance if any two of them agree, the whole will be very inconsistent with itself, and, therefore, must make a very awkward appearance.

This

This consideration, I think, is a matter of consequence for the encouragement of young designers. I shall, therefore, give them an example of it, in my own practice.

The drawing, plate vii, which is a perspective view of an electrical machine of my construction, was made when I had very little knowledge of the theory or practice of perspective. I only knew how to fix vanishing points in the horizontal line, according to any position of lines upon the ground plane; but as all the planes in this machine were either perpendicular to the ground plane, or parallel to it, I was sensible that I wanted no other; and though I knew but little of the doctrine of perspective measures, I could judge nearly enough of the proportion of the several lines by my eye, and therefore could do without them.

All that I did, therefore, was to consider the center of the globe as the point of sight, and to draw the horizontal line and perpendicular crossing one another at right angles in it. On the perpendicular
I set

I set off the distance at which I chose to work, and from the same point set off angles of 35 degrees on one side, and 55 on the other, which gave me two vanishing points, from which I could draw lines cutting one another at right angles. Then observing the angle under which the whole machine was viewed in the situation I chose for myself, and having the machine before me, I made every thing as nearly in proportion to it as I could judge by my eye. As I measured nothing, I drew no ground line, but, when I had done, closed the drawing where I thought proper. It is a fault in this drawing, that I fixed the point of distance too near the point of sight. But this doth not make the drawing less just, or less useful.

The electrical battery, plate iii, was drawn at the same time. Both the sides have equal degrees of obliquity to the horizontal line; and therefore I drew all the lines to one or other of the points of distance (or the tangents of 45 degrees) on each side of the point of sight. Here, also, I had no measures, or ground line.

With

With more knowledge of the art of perspective I might have made more elegant drawings of these figures, but these are perfectly just, and sufficiently plain. In these, and in many other cases, I have often been agreeably surprized to find how much useful practical knowledge may be derived from a very little theory.

I could engage to communicate to any person all the knowledge that is requisite to make these two drawings in two or three minutes, whereby he might finish them, at his leisure, in little more than an hour each; and without that little knowledge, a person might puzzle himself a month about them, and, after all, produce nothing that should not be quite preposterous.

It is a maxim with designers, founded upon experience, that no picture should take in quite so much as is comprehended within an angle of 90 degrees. In other words, the figure should not extend to both the points of distance on the horizontal line. The reason of this rule is, that the eye cannot, at one time, distinctly take in so
great

great a compass, but the extremities of the prospect will be confused and indistinct.

In general, the least distance should be about one-third more than the height of the eye. In many drawings it should be much greater.

In drawing landscapes, a low horizon is particularly agreeable. It should be about one-third of the depth of the picture.

The rule laid down above for chusing the distance, should be observed even where very high horizons are chosen, in order to give what is commonly called *a bird's view* of objects. Thus [a] Fig. 47, is a view of a plain square inclosure, with a pretty low horizon, and [b] is a view of the same square inclosure with a higher horizon, and a proportionably greater distance. By this means more of the inside of the inclosure is seen.

I shall

I shall close this part of the work with the following extracts from Mr. Emerson's Perspective.

“ If any print or perspective view, be looked at through a lens, whose focal distance is equal to the principal ray, and the print placed in its focus; it will be so magnified, as to have the very same appearance, as at the place it was drawn for.

“ Glasses which are not of a due focal length, will not give the exact appearance of a place. Shorter glasses make the distances less, and so contract the view. And longer glasses make the distances greater, and extend the view.

“ Since glasses of a long focal distance, give a large and extensive prospect of a country, therefore they are better than shorter glasses. And when the prospects are well drawn, and properly coloured; it is very delightful to view them through a good glass, as they so nearly imitate nature. And though there is but one focal length
that

that will give a true appearance, yet the draught will always appear a regular piece of perspective, though it may not exactly represent any place in the world, supposing the eye placed somewhere in the principal ray. And the draught will seem longer in proportion to the focal distance of the glass made use of; or in proportion to the apparent distance of the nearest part of the picture.

“ As there is nothing more pleasant than viewing the draughts of countries, towns, cities, magnificent buildings, and other grand objects, when well drawn: to see them to the best advantage, the focal distance of the glass should be just so long, as not to shew the scratches and coarseness of the engraving; or not much longer; for then the view will be narrow, and the parts too small to be seen so far off. And if it be far off, it will hide the beauties as well as deformities. And to get a proper glass, observe at what distance the scratches disappear to the naked eye, and that is the focal length of the glass. Perspective views should be drawn, so that the point of view
be

be farther off, than is generally practised, if you would have the piece to be a true copy of nature. The principal ray should not be less than two feet, and then the draught, being looked at through a lens of that focal distance, will appear in perfection, and give a true representation of the place it was drawn for. The view should be so large as to subtend an angle at the lens of about 30 degrees. And it is proper to put the lens in a short square tube; which will confine the sight, and direct it to the perspective draught; which, to complete its beauty, ought to be coloured with the same colours as the natural objects appear in. For this purpose water colours need only be used."



The definition of all the technical terms made use of in this treatise.

THE *ground plane*; the plane on which both the spectator, and the objects that are to be drawn, stand.

The *perspective plane*; a plane standing perpendicularly upon the ground plane, interposed between the eye and the objects. On this plane (as on a glass window) the images of objects are supposed to be intercepted; so that their *perspective appearance* is the appearance they have on this plane.

The *ground line*; the line on which the perspective plane is supposed to rest.

The *point of sight*; that point in the perspective plane which is nearest to the eye, and at the same distance from the ground line with the height of the eye above the ground plane. N. B. A line drawn from the eye to the point of sight is sometimes called the *principal ray*.

The

The *horizontal line*; a line upon the perspective plane, drawn through the point of sight, parallel to the ground line.

The *perpendicular*; a line on the perspective plane, drawn through the point of sight, perpendicular to the ground line and the horizontal line.

Points of distance; points on the perspective plane, set off from the point of sight, sometimes on the horizontal line, sometimes on the perpendicular; at the same distance from the point of sight that the eye is supposed to be at from the perspective plane.

Vanishing points; points on the perspective plane, in which parallel lines, infinitely produced, seem to meet.

Measuring points; points from which any lines in the perspective plane are measured, by laying a ruler from them to the divisions laid down upon the ground line.

REFLECTIVE

The horizontal line is upon the positive plane, drawn through the point of light parallel to the ground line.

The horizontal line is upon the positive plane, drawn through the point of light, perpendicular to the ground line and the horizontal line.

Point of light: point on the positive plane, for the horizontal line of light, perpendicular to the ground line, and the horizontal line, is the point of light, and the distance from the point of light to the horizontal line is the distance from the point of light to the positive plane.

Horizontal line: point on the positive plane, in which parallel line is drawn, and the horizontal line is the horizontal line.

Horizontal line: point on the positive plane, in which parallel line is drawn, and the horizontal line is the horizontal line, and the horizontal line is the horizontal line.

A
GENERAL VIEW
OF
THE THEORY OF
PERSPECTIVE.

WITH
NOTES,
RELATING TO IT.

G 3

A

GENERAL VIEW

THE THEORY OF

PERSPECTIVE

WITH

NUMEROUS

ILLUSTRATIONS



A GENERAL VIEW of
THE THEORY OF
PERSPECTIVE,

To which some of the particular DEMONSTRATIONS in the following NOTES refer.

DEFINITION I.

The vanishing line of an original plane, is a right line in the picture, formed by the intersection of an imaginary plane passing through the eye, and parallel to the original one, with the picture.

THUS, in Fig. 4, having raised the planes Y and Z, if we suppose a plane to go through the eye at *a*, and to be parallel to the ground plane X, that plane will

will cut the picture Y, in the horizontal line FG; which, therefore, is the vanishing line of the plane X, or ground plane.

A little attention will make it evident, that no point in the original plane X, can have its image in the picture, above the line FG, so long as the position of the picture and the eye remain the same; and, that if the plane X were infinitely extended beyond the picture, yet its perspective appearance, in the present example, would be a finite space, bounded by the vanishing line FG.

DEFINITION II.

The vanishing point of an original line, is that point in the picture, where an imaginary line, drawn from the eye, parallel to the given line, cuts it.

THE planes Y and Z, Fig. 4, being raised as before, the vanishing point of the line CD, on the plane X, perpendicular to the picture, will be the point D; for it is plain, that an imaginary line drawn from

from the eye a , to D , is parallel to the line CD , since $CD = ba$, and the line aD is also perpendicular to the picture.

It will not be difficult to conceive, that were the original line CD , extended beyond the picture, to any distance whatever, the image of no point in it could appear higher in the picture, than the point D .

P R O P O S I T I O N I.

Supposing the picture a plane, which may be extended every way at pleasure, the line in which any original plane, not parallel to the picture, cuts the picture (and which may be called the intersecting line of that plane,) is parallel to the vanishing line of the same plane.

FOR, since the imaginary plane, passing through the eye, and producing the vanishing line, is parallel to the original plane, the lines formed by their intersections with any other plane, as the picture, will be parallel (16 Eu. 11.) Thus the ground line AB , Fig. 4, which is the intersecting

intersecting line of the ground plane, is parallel to the horizontal, or vanishing line of that plane.

Corollary. Planes parallel to the picture, can have no vanishing line.

For a plane passing through the eye parallel to such planes, is also parallel to the picture, and therefore can never cut it.

PROPOSITION II.

The image of every original line, perpendicular, or oblique to the picture, tends to its vanishing point: and the images of such original lines, as are parallel to the picture, can have no vanishing points; they will, therefore, be parallel to their respective originals.

LET the planes, Fig 4, be prepared as before; and let any original line, as KM, be produced till it cut the picture in C (which point is called the *intersecting point* of the line KM) join this point and the vanishing point D, of the line; then, the appearance of every point in the original line, infinitely extended beyond C, will

will be found in this line, and consequently, the image *km* of any part *KM* thereof, must tend to the point *D*.

A line parallel to the picture, will have the imaginary line drawn from the eye, which should find its vanishing point, parallel to the picture also; so that such a line can have no vanishing point. The image of such a line will, therefore, be parallel to this imaginary line drawn from the eye, and, of consequence, parallel to its original.

Corollary. The images of parallel lines, not parallel to the picture, tend to the same point; and parallel lines, parallel also to the picture, will have their images parallel one to another.

P R O P O S I T I O N III.

The vanishing point of every original line, lying in the same original plane, will be found in the vanishing line of that plane.

FOR the vanishing line of a plane, is the boundary of the appearance of that plane;

plane, infinitely extended before the eye; and the vanishing point of a line, is also the boundary of the image of that line, extended in like manner. It is plain, therefore, that the latter must be found somewhere in the former.

So the vanishing point of the line KM, Fig. 4, is a point D, in the vanishing line FG, of the plane X, in which it lies.

Corollary I. Hence, if we know the vanishing points of two lines, not parallel the one to the other, and which lie in the same plane; we know, also, the vanishing line of that plane.

For a line joining those vanishing points, is the vanishing line required.

Corollary II. The imaginary line drawn from the eye, which finds the vanishing point of any original line, is parallel to the plane in which that line lies.

PROPOSITION IV.

The image of a line, parallel to the picture, will be similarly posited with its object; and have its length in proportion to that of its original, as the distance between the eye and the image, is to the distance between the eye and the object.

LET Y, Fig. 48, be the picture, E the place of the eye, AB an original line, parallel to the picture; I say, $ab : AB :: Eb : EB$.

For, ab being parallel to AB, (Prop. II.) the triangles Eab and EAB are similar; whence, $ab : AB :: Eb : EB :: Ea : EA$.

PROPO-

PROPOSITION V.

The image of any point, in a given original line, not parallel to the picture, is a point in the indefinite image of that line, which divides the said indefinite image in the proportion of the distance between the eye and vanishing point of the given line, to the distance between the given point and point of intersection.

LET Y, Fig. 49, be the perspective plane, E the place of the eye, AB an original line, whose vanishing and intersecting points are *a* and A, respectively; I say, $ab : bA :: Ea : AB$.

For, since *Ea* and AB are parallel, (Def. II.) the triangles *Eab* and *BAb* are similar; whence $ab : bA :: Ea : AB$.



N O T E S,



Relating, chiefly, to

THE THEORY OF P E R S P E C T I V E.

NOTE A. Page 8.

IN drawing the few objects comprised in Fig. 2, there is occasion to introduce every real variety in the practice of perspective. It will, therefore, be of use to the learner to attend to it; and when he is master of all the necessary rules, to draw it over frequently, varying the lengths and situations of the lines contained in it. For when once a person can draw any line in this small picture, independent of the rest, from the situation and measure of it, previously given, he will be able to draw the representation

resentation of any object, or any number of objects whatever, be they ever so complex,

B. Page 14.

Since these lines are parallel to the ground line, they are parallel to the picture; hence, (per Prop. II.) their images will be parallel to the original, and, therefore, must be parallel to the ground line also.

This is clearly exemplified in Fig. 4, where *OP* and *HI*, upon the ground plane *X*, are represented by *op* and *hi* on the perspective plane *Y*. For if the perspective plane be perforated in *k*, *l*, *m* and *n*, and a straight wire be put through *a* in the plate *Z*, representing the eye of the observer, and also through the holes *k* or *l*, it will fall upon *OP*; and if it be put through the holes *m* and *n*, it will fall upon the lines *HI*.

C. Page 15.

That this method of determining the distances of these parallel lines must be accurate,

curate, will be evident from considering that G , Fig. 4, being situated at the height of the eye from the ground line AB , and GD being the distance of the eye from the perspective plane, the line DC may be imagined to be the perspective plane, seen edgeways; and then any point q situated three measures beyond it, must appear at m , in a straight line drawn from the eye at G to it: m , therefore, is the proper distance from C , at which that point in the line required must be drawn. Now, supposing the perspective plane to revolve, and to be restored to its former situation, with the eye at a ; for the same reason that m must be placed at the distance of mC from the ground line AB , every point in the same line must be placed at the same distance; and therefore a parallel to the ground line, drawn through m , will be the line required.

To explain this more clearly, put wires through a and m , when the planes Y and Z are set upright; and it will be seen, that this wire is exactly in the same situation, and of the same length, with the line Gq ;

H

both

both of them passing through m , and cutting of the same length, Cm , from the line CD .

D. Page 16.

The more of these lines I draw upon the ground plane, equidistant from one another, the nearer they seem to approach, when reduced to the perspective plane; because every interval between them is continually farther from the eye. The lines CA , Fig. 5, being continued *ad infinitum*, and the same process repeated, it is plain that every succeeding parallel would approach nearer to the preceding one, till at last they coincided with the horizontal line FG .

E. Page 18.

That all such lines, perpendicular to the ground line, will meet on the perspective plane in the point of sight D , is also evident from considering the situation of the space included between the parallel lines QD and SD , each of which is equally distant from the perpendicular CD . The extremity of this space QS , being nearest
to

to the eye, must appear wider than the distance between these lines at any place more remote from the eye; and the farther off these distances are taken, the less they will appear. Consequently the sides which bound this space must appear to approach nearer and nearer continually, till they meet somewhere; and this point, in which they meet or vanish, must be somewhere in the line CD ; because each side of the space is every where equidistant from the eye.

It must, likewise, be somewhere in the horizontal line FG ; for raising the planes Y and Z , the point M on the ground plane will appear in m on the perspective plane, to an eye situated at a ; the point K will appear in k ; and so CD on the ground plane, being infinitely continued, and every point in it, more remote from the eye, appearing to rise higher and higher in the perspective plane, they must at last reach as high as D . And they can never rise higher than D ; because, where the visual ray and the line CD actually parallel, still the former could only pass through D .

For

For the same reason that the lines QD and SD will meet at D on the perspective plane, the representations of RD, TD, and all other lines that are parallel to them, at whatever distance they be taken, on either side of the perpendicular CD, must meet in the same point of sight D.

This is evident from Def. II. for the original lines being perpendicular to the ground line, are perpendicular to the picture; therefore, the imaginary line passing through the eye, which gives the vanishing point of these lines, will be perpendicular to the picture also; but it must be parallel to the ground plane (Cor. II. Prop. III.) consequently will cut the picture in the point of sight.

F. Page 19.

It is evident, that CI, Fig. 6, being situated upon the ground plane, must have its vanishing point somewhere in the horizontal line DF, for the reason given before Prop. V. It will also be that point in the
hori-

horizontal line where an imaginary line drawn from the eye, parallel to CI , cuts it (Def. II.) This imaginary line will, therefore, form an angle, at the eye a , with the perpendicular aD , equal the angle ICD . The part Di intercepted between D , and this vanishing point must, consequently, be the *tangent* to that angle, the radius being aD ; or, which is the same thing, the radius being DE .

Let the plane Y be interposed between the eye at a and the line CD upon the ground plane, and it will be evident that the base of the infinite triangle (formed by the lines CD and CI) in the true horizon, and the line Di upon the perspective plane, which represents it, are equally tangents to the same angle, under which they are both viewed; all the difference between them being in the radius of the circle.

Wherever Ci meets the horizontal line, all parallels to it must be drawn to the same point, for the reason given before (Cor. I, Prop. II.)

To illustrate the preceding demonstration by experiment, let a straight wire pass from a to H , Fig. 6, and it will pass through the perspective plane Y in b . If a more distant point in the line CI be taken, the plane Y will be pierced in the line Ch produced, which shows that it tends to vanish in i .

For the same reason that the vanishing point of Ci , and of all its parallels, is in the horizontal line FG , the vanishing points of all lines situated upon the ground plane, in whatever direction they be drawn, must be somewhere in the same line. For this reason the horizontal line is called the *vanishing line* of the ground plane.

And for the same reason that the ground plane hath its vanishing line in the horizon, any other plane, making any angle with the ground plane, will have a vanishing line, making the same angle with the horizontal line, which the intersecting line of the new plane makes with the ground line; and every operation may be performed with this, as with the first ground line, and the
first

first horizon. This consideration throws the greatest clearness into the theory of perspective, and makes the practice exceeding easy; and this great improvement we owe to the sagacity of Dr. Brook Taylor.

G. Page 21.

Since lines perpendicular to the ground plane are parallel to the picture, their images must be parallel one to another and to their originals. (Prop. II.) Those images will, consequently, be perpendicular to the ground line.

To illustrate this, let pins, and other bodies, be fixed upright on the ground plane X, and considered as viewed from a in Z, while the representation is taken upon the plane Y.

H. Page 22.

Imagine the given line CK to be situated in a plane [x] perpendicular to the ground plane, such a plane will cut the ground plane in the line CH, making the angle
DCH

DCH equal the given declination of the line CK; the point i is, therefore, the vanishing point of CH; and, since CH lies in the plane $[x]$ as well as in the plane X, the vanishing line of the plane $[x]$ will pass through the point i (Prop. III.) Again, the plane $[x]$ being perpendicular to the ground plane, would, if extended, intersect the picture in the line CDE; hence its vanishing line will be parallel to DE (Prop. I.) therefore the vanishing point of CK will be found in ig . We are, lastly, to prove that g is that vanishing point.

Since the imaginary line ag is parallel to CK (Def. II.) it must make the angle iag = the angle HCK, or angle of elevation of the given line. The line ig is, therefore, the tangent of the angle HCK to radius equal ai . But ig is the tangent of the angle iyg (= HCK) to radius iy and $iy = iE = ia$. Consequently, the point g is truly determined.

The imagination may be assisted in conceiving of this, by supposing the horizon to be raised as much as the angle of elevation

tion

tion of the given line. For, in this case, the representation Ck of the line produced, must as certainly meet this new horizontal line, passing through g , as any lines upon the ground plane, continued, must meet the common horizon. Indeed, upon this supposition, a plane passing through CK , and the foot of the observer, would take place of the old ground plane.

I. Page 26.

Raise the planes Y and Z , Fig. 4. Then, since the given line HI is parallel to the picture, the image un of any part UN thereof, will be parallel to its original; whence per Prop. IV. $un : UN :: au : aU$; $un : (Cs =) UN :: Gu : GC$, per construction; but since $DG = aD$, $Gu = au$; and $GC = aU$, therefore the image un is justly determined, by means of the lines GC , Gs . Again, we have $un : (st = UN) :: Du : Ds$; but $Du : Ds :: Gu : GC$; hence, un is the image of UN , as found by this method also.

By

By raising the moveable plates in Fig. 4, and putting a straight wire through a , and the several perforations in the perspective plane Y , these rules will be occularly demonstrated. For a straight wire extended from a to U , will pass through u , and being extended to M it will pass through m , so as to intercept um in the perspective plane, corresponding to UM on the ground plane. And since triangles standing upon equal bases and between the same parallels are equal, the triangle aUN will be equal to the triangle GCs , or Dst , a being the same height from the ground plane with D and G . Also if these equal triangles be cut by a plane parallel to their bases, as by the plane hi , the new bases will still be equal; so that un , in this case, will be the base to both the triangles. For the same reason, they would have been as well divided by lines drawn from F , the other point of distance, as from G .

Universally, perspective measures are determined by drawing lines from the points of distance, or other measuring points, to the measures, previously laid down upon the ground plane, or to lines situated in a
fimilar

similar manner with regard to the perspective plane.

K. Page 27.

Supposing a line to be drawn from a to M , it will pass through the perspective plane in m . But Cq being equal to CM , and DG equal to aD , the triangles aCM and GqC will be exactly equal; the latter being, in fact, the very same with the former, when the lines aD and aM have been supposed to revolve, till a came into the situation of G . Consequently m being an immovable point during the revolution, its place will be the very same in which ever of the triangles it be taken.

In the very same manner may it be shown, that any other point, as u or n , will be found to have the same place upon the perspective plane, whether it be fixed by a line drawn through it from the eye to the object, or by a line drawn through it from the point of distance in the horizon to a point on the ground line, at the same distance from the perpendicular with the object itself. For by Prop. V. we have $Du :$

us

$us :: (aD =) DG : SU$; but from the similarity of the triangles DuG and suC , it is, as Du is to us , so is DG to Cs ; but $Cs = SU$ per construction. Therefore the point u is the image of U . The same reasoning will prove, the point n to be the image of N .

L. Page 29.

Since the triangles iyb and bCb are similar, we have $ib : bC :: iy : Cb$. But $Cb = CH$ and $iy = iE = ia$ per construction. Therefore $ib : bC :: ia : CH$; whence per Prop. V. the point b is the image of H , as was required.

M. Page 33.

The process in this section hardly needs any demonstration, it being evident that any row of objects, of the same height, standing upon the ground plane, on a line perpendicular or oblique to the ground line, must appear to diminish till they vanish in the horizontal line. Consequently the triangle dfe , Fig. 15, will contain in it perpendicular lines equal to the apparent height

height of all objects that are of the height of fe ; and this will be the case wherever the point d is taken in the horizontal line. For $ib : ef :: di : de$; but $di : de :: dm : dk$, that is, as $Ha : HI$; and since this proportion will hold wherever the point d be taken in the horizontal line, it follows, that the length of the perpendicular ib will not be altered thereby; it being always to ef in the constant proportion of Ha to HI .

N. Page 34.

In the first method, we have the triangles kgy and kCa similar; whence $gk : kC :: gy : Ca$; but gy is equal to ga , the distance of the eye from the vanishing point g , and Ca is equal CK per construction. Therefore $gk : kC :: ga : CK$. Hence (per Prop V.) the point k is the image of K . In the second method, since gy is drawn parallel to Ca , we have $gk : kC :: gy : Ca$; but gy and Ca are equal gy and Ca , and of consequence equal ga and CK respectively. Therefore, &c. as before.

O.



Let AB, Fig. 21, be the original of ab , on which describe the circle AIB IIII, and draw the diameters I, IIII; LIII, IIV, dividing the circle into 8 equal parts. Then it is plain, the diameters I IIII, V II, L III, will have their vanishing points at C, D, G; and therefore $1C$, $2D$, and lG are their indefinite images; and since AI and IIII B are parallel to V II, they will have the same vanishing point with V II, viz. D. Therefore, lines drawn from D to a and b will cut $1C$ in 1 and 4, the images of I and IIII.

Since F is the measuring point of lG , and H the measuring point of $2sD$, lines drawn from F to a and b will cut off the parts ls , $s3$ respectively, equal to as and sb . For the same reason, lines drawn from H to a and b will cut off the parts $s5$ $s2$ equal as , sb . Consequently, $l3$, and 52 are the images of L III and V II, diameters of the original circle AIB IIII. Q. E. D.

P.

P. Page 55.

The demonstration of this rule is made easy by the consideration of Fig. 6. Let HC, in the plane X, be the line to be determined. The vanishing point is fixed by the line *ai*, drawn parallel to HC, or *Ei*, which, making the same angle with the perpendicular, meets the horizontal line in the same place *i*. And the point *b* is determined by drawing a line from H to *a*, the place of the eye. In both these cases the picture is placed between the eye and the object; and if the elevation of the eye, in Fig. 6, occasion any difficulty to the person who compares these figures, let him lay the plane Y flat down, so as to be a continuation of the plane X, and let him suppose E and [h] to be visible through it on the backside of the paper (as they are marked in the figure for that purpose) and he will find that a ruler laid from H to E, will pass through the point *b*. In this situation of the plates X and Y, all the lines have the very same position to one another as the lines in Fig. 23.

Q.

Q. Page 61.

The *rationale* of this practice is very obvious, depending on this one principle, that every line in the perspective plane makes a tangent to the angle of vision; and it may be fully illustrated by the help of Fig. 6. Let g , in the plane Y , be the point in the perspective plane, representing the point of any object that is required to be drawn. When the planes Z and Y are raised, it will be evident that the angle iED is equal to iaD . Consequently i , the foot of the perpendicular ig , is truly fixed by this rule. Also the angle iyg is equal to iag , so that the point g is likewise truly fixed by it.

R. Page 70.

To demonstrate this case, I shall make use of a larger figure, Fig. 50.

Let the triangle to be projected be Cbi in the plane Y , equal to CBI in the plane X . Proceed as in the former example, and both of the triangles will be completed

pleted by producing the sides bi , and BI to A . If the visual rays be perpendicular to the perspective plane Y , and the triangle $A\hat{b}C$ be made to turn upon its base AC , as in the plane X , it is evident that, in every position of the plane X , the shadow or image of the point B will always fall upon some part of the line $\hat{b}D$, and, at whatever angle the eye be elevated above the plane X (suppose 30 degrees) it is also evident, that (raising the plane Z) the perpendicular BH will fall upon the same point H , in the line $\hat{b}D$, as the parallel GH (FG being taken = 30 degrees = DBH) for since the triangles BDH and GDH have BD equal to GD , DBH equal to DGH , and the angles at H right ones, the whole triangles will be equal; and consequently the line DH will be the same in both; $A\hat{b}$ will, therefore, be projected into AH , $C\hat{b}$ into HC , and the angle $A\hat{b}C$ into AHC .

If the side $\hat{b}A$ terminate in i , and BA in I ; draw $i\hat{k}$ parallel to $\hat{b}D$, and it will cut HA in K , the place of the image of I .

I

It

It is evident from what was advanced before, that the image of the point [I] must fall somewhere in the line HA: It is equally evident, that it must fall somewhere in the perpendicular *ik*, in every position of the plane X; and therefore can be no where but in the intersection, K, of these two lines.

An Addition to PART XII. p. 94.

THAT the young artist may be a proper judge of the importance of chusing a proper *distance* in perspective drawings, I have given a duplicate of Fig. 20. in Fig. 51. In order to take in all the lines and points that were principally requisite to the description, the distance, DE, in the former, was taken very small; in consequence of which, the more remote squares seem to be out of proportion, though to an eye in a proper situation for viewing them (*viz.* at the distance DE, perpendicularly over the point D) the angles they subtend will be found

found to be equal, and therefore the drawing is just. Fig. 51 differs from the former in nothing but the distance, which is taken near three times as great. By this means all the squares fall within the measuring points *a* and *b*, and the distortion, observable in the former figure, is avoided.

What we call the perspective plane is only an imaginary thing; for, in reality, we do not refer the objects of sight to any plane, some parts of which are more distant from the eye than others. Whenever, therefore, the distance in drawing is so small, that one part of the picture is considerably more remote than another, a distortion will be perceived by an eye in the situation in which drawings are generally viewed. No drawing can appear perfectly natural except on the surface of a sphere, the eye being placed in the center. But this inconvenience, attending the common perspective, will be sufficiently obviated, if the distance be taken large, in proportion to the size of the figures to be drawn. It is because these things are better understood
by

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THAT the young artist may be a proper judge of the importance of chusing a proper *distance* in perspective drawings, I have given a duplicate of Fig. 20. in Fig. 51. In order to take in all the lines and points that were principally requisite to the description, the distance, DE, in the former, was taken very small; in consequence of which, the more remote squares seem to be out of proportion, though to an eye in a proper situation for viewing them (*viz.* at the distance DE, perpendicularly over the point D) the angles they subtend will be found

found to be equal, and therefore the drawing is just. Fig. 51 differs from the former in nothing but the distance, which is taken near three times as great. By this means all the squares fall within the measuring points *a* and *b*, and the distortion, observable in the former figure, is avoided.

What we call the perspective plane is only an imaginary thing; for, in reality, we do not refer the objects of sight to any plane, some parts of which are more distant from the eye than others. Whenever, therefore, the distance in drawing is so small, that one part of the picture is considerably more remote than another, a distortion will be perceived by an eye in the situation in which drawings are generally viewed. No drawing can appear perfectly natural except on the surface of a sphere, the eye being placed in the center. But this inconvenience, attending the common perspective, will be sufficiently obviated, if the distance be taken large, in proportion to the size of the figures to be drawn. It is because these things are better understood
by

by example than precept that I have given the two drawings of these figures.

Upon the principle mentioned above, the problem concerning the perspective diameter of columns, standing at equal distances from the ground line, admits of a solution; but it is not to my purpose in this treatise to discuss such particulars.



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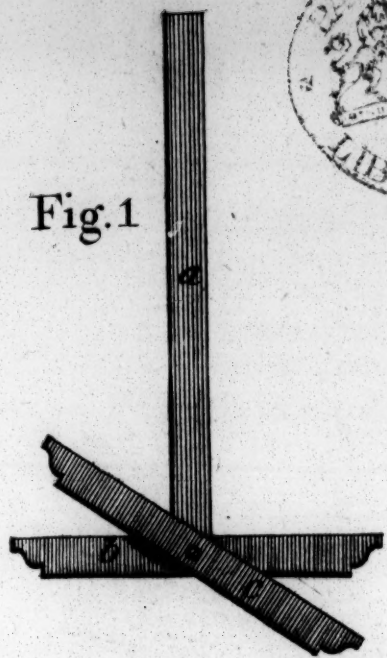


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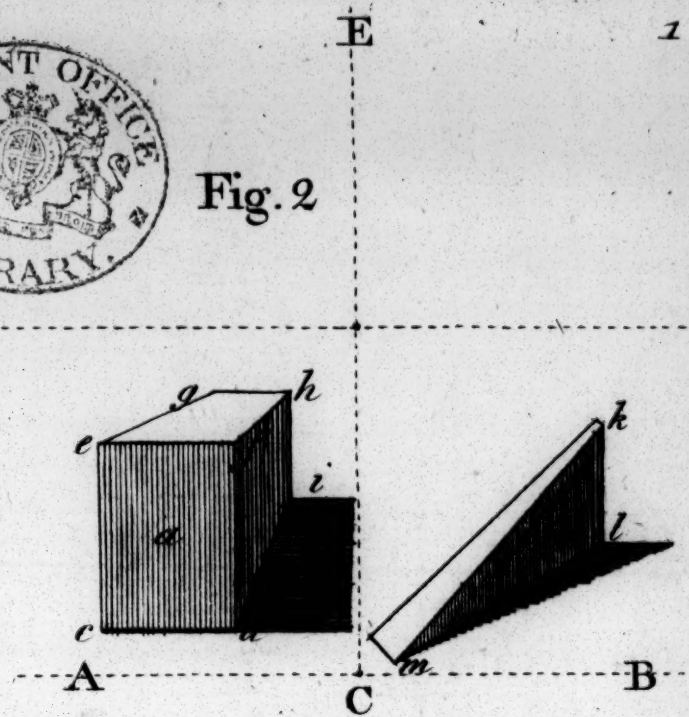
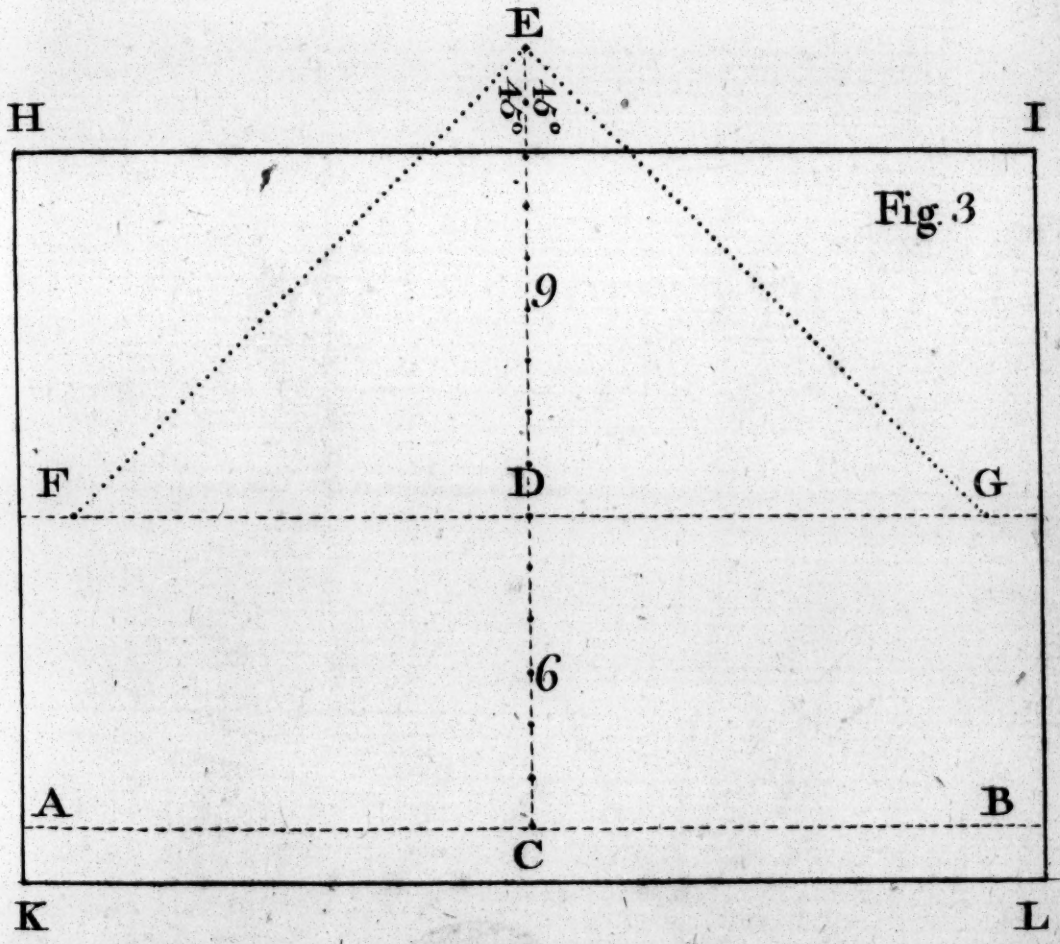


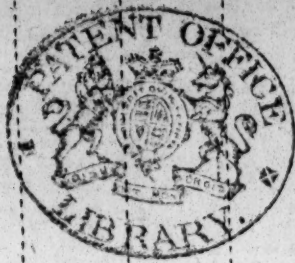
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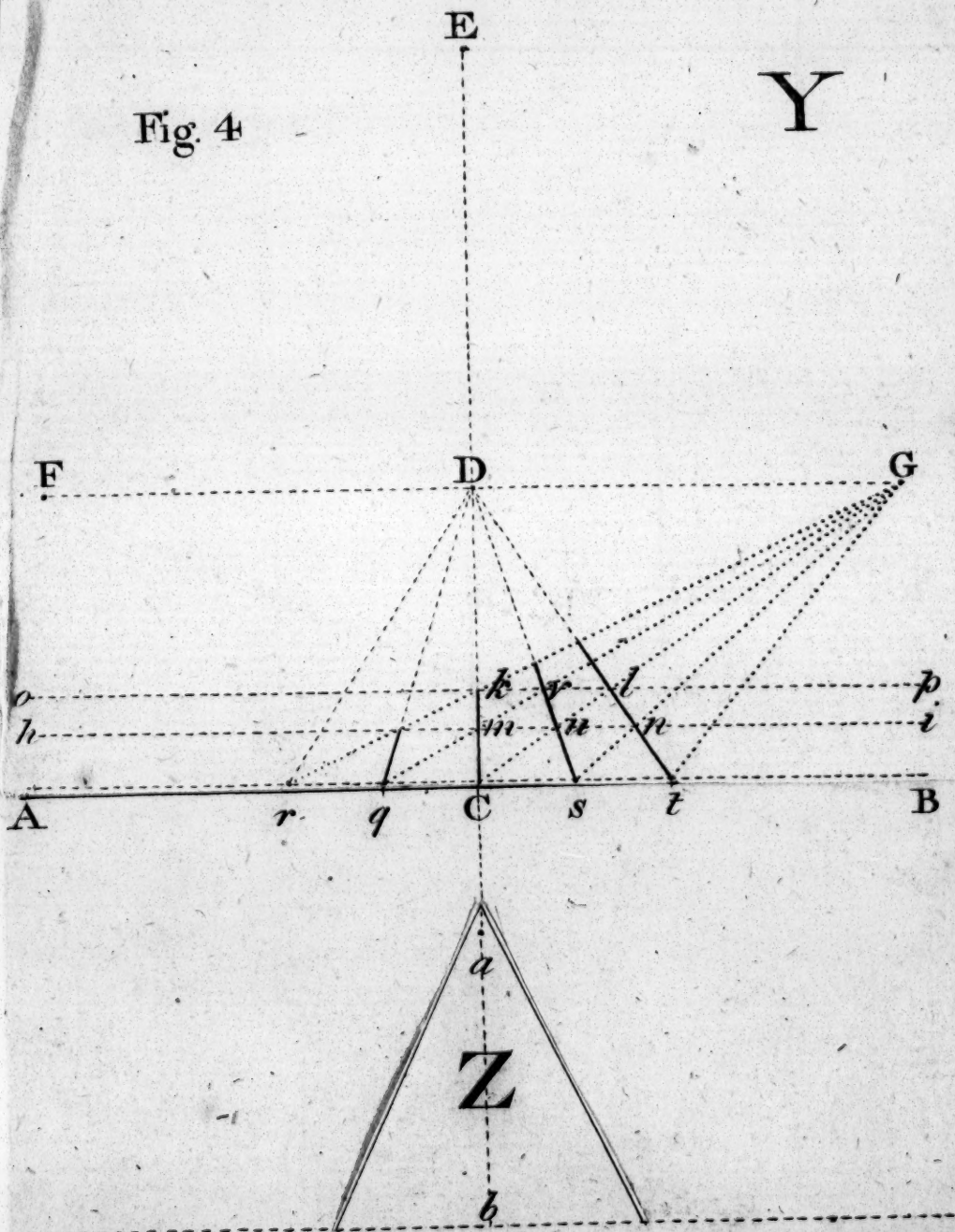


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Fig. 4

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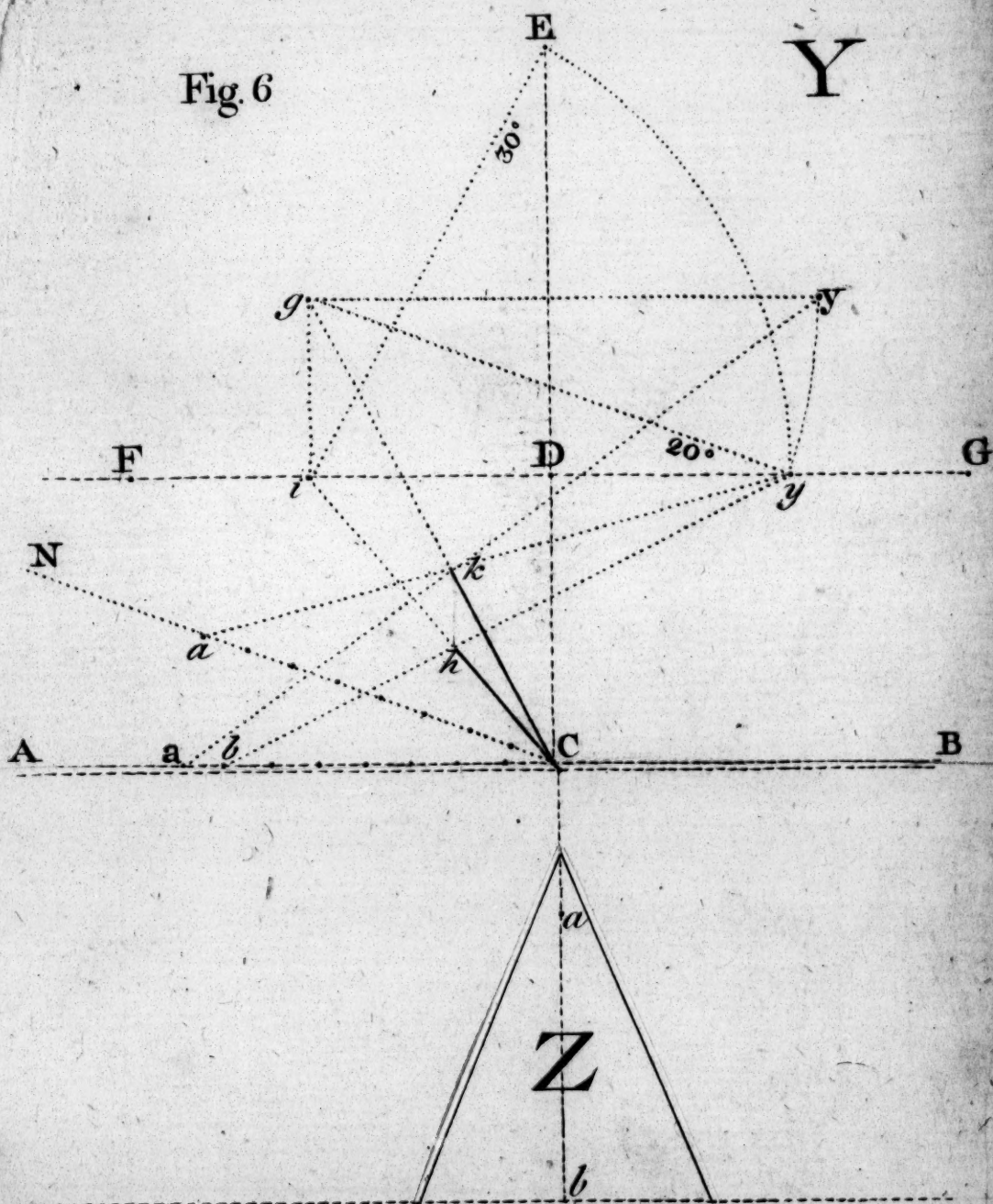
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Fig. 6.

Fig. 6



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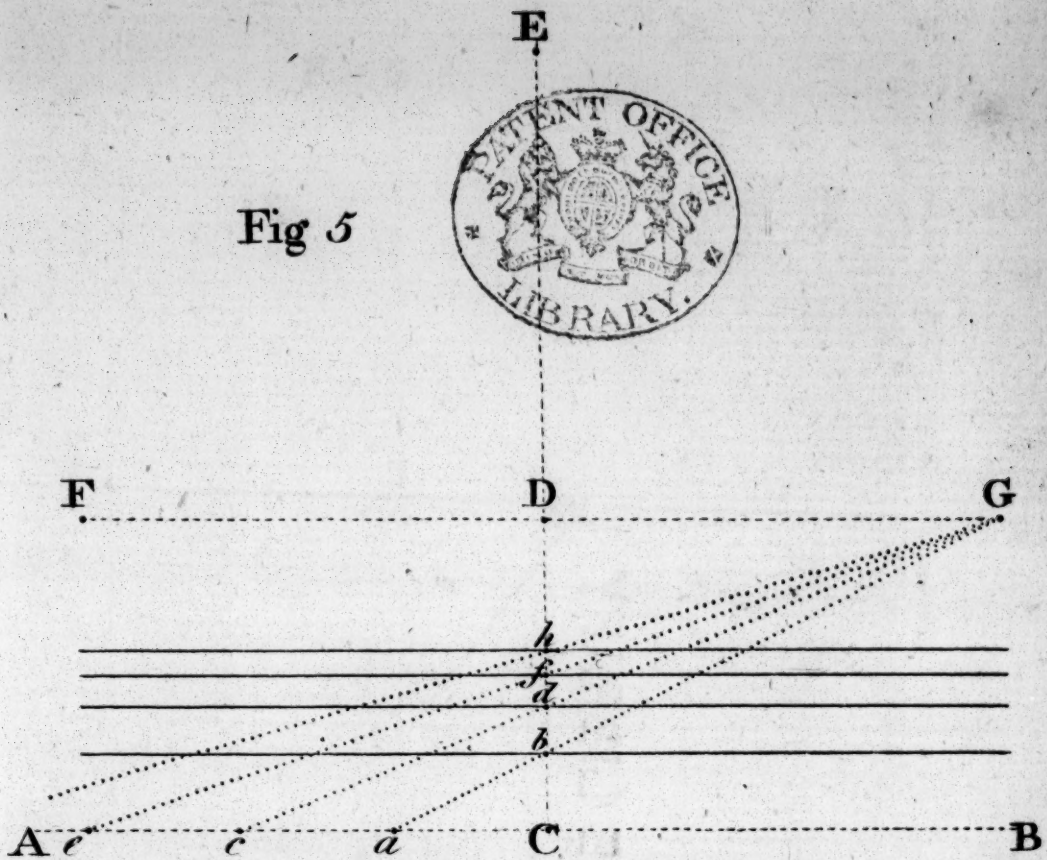
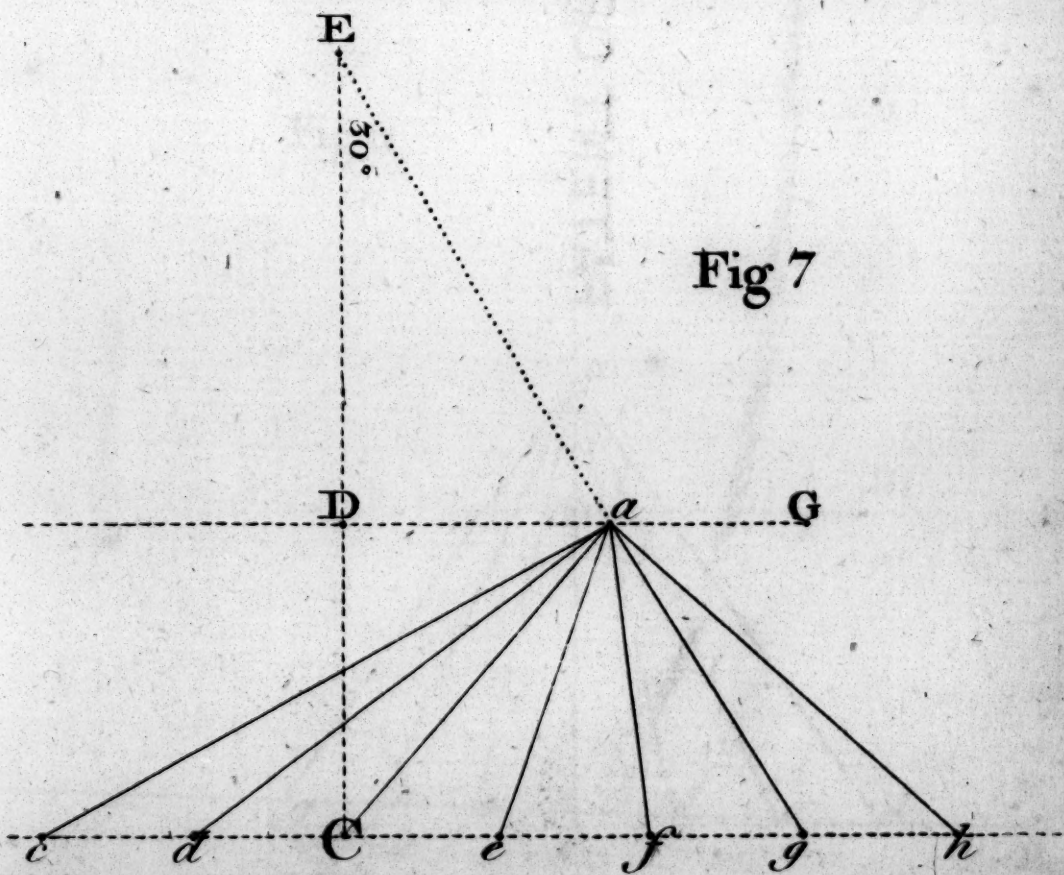


Fig 7



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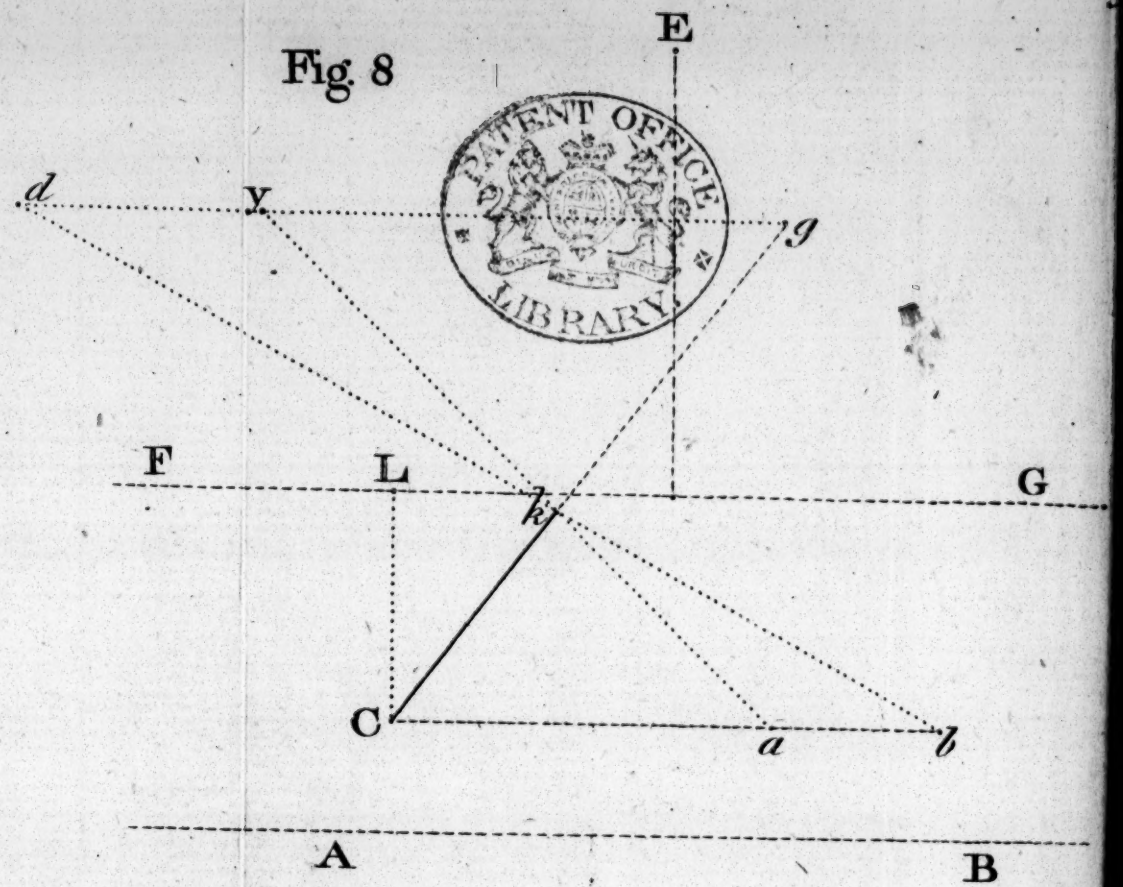
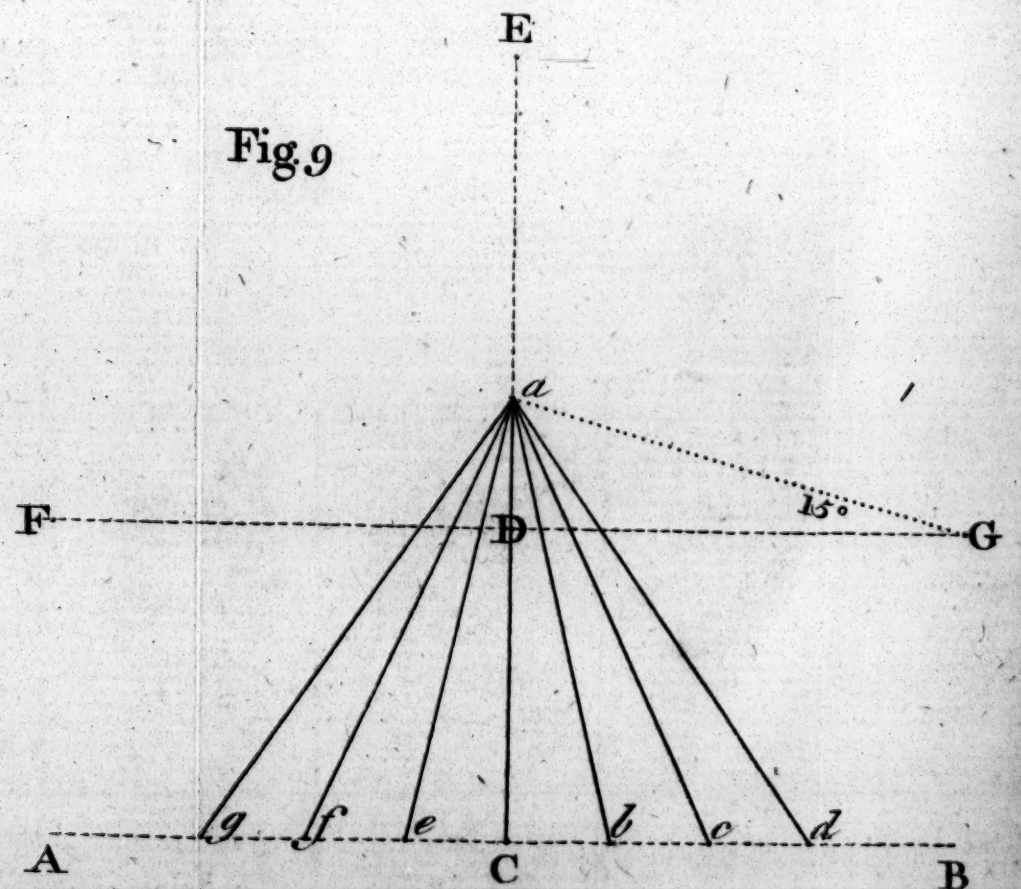
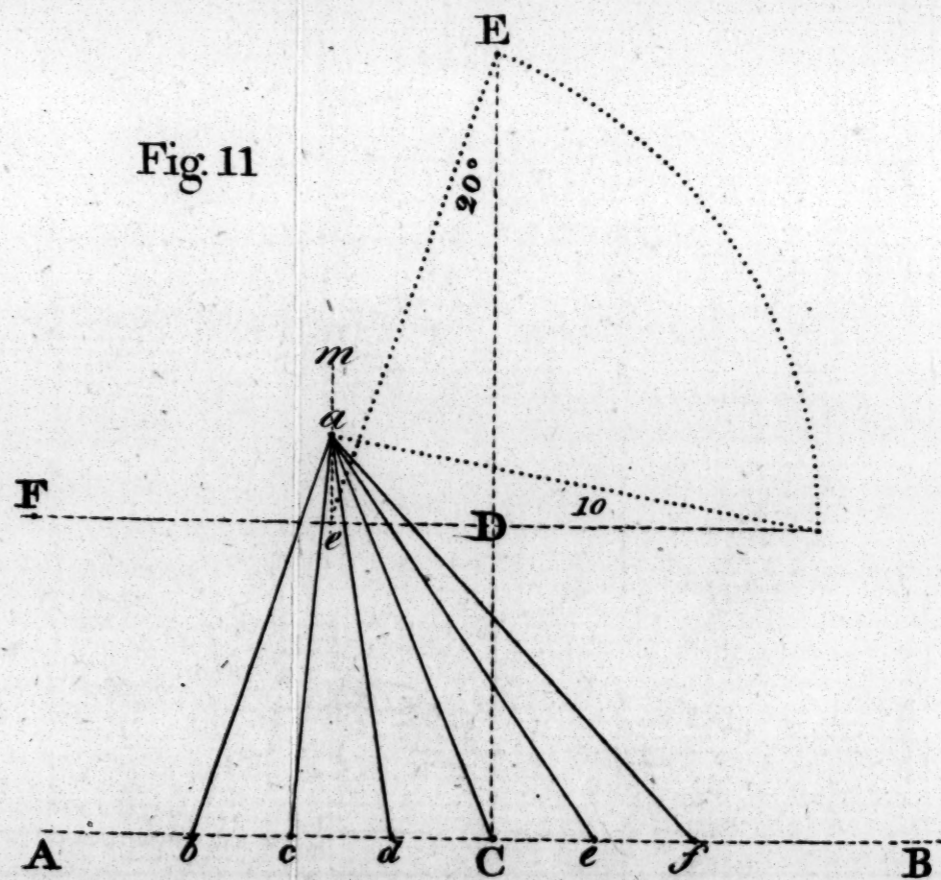
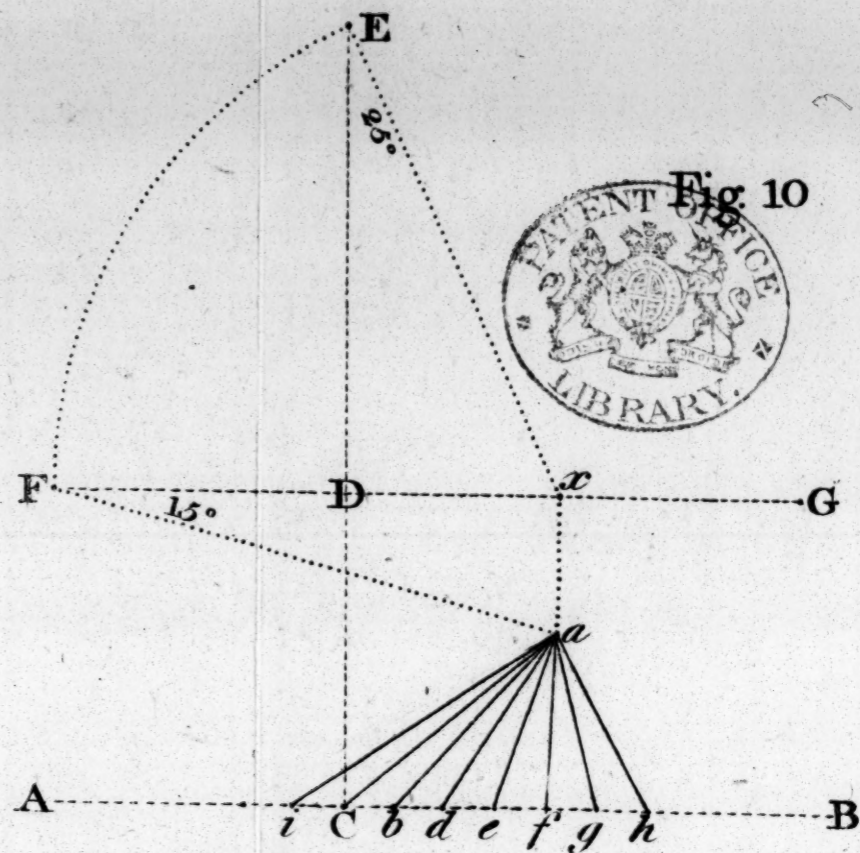


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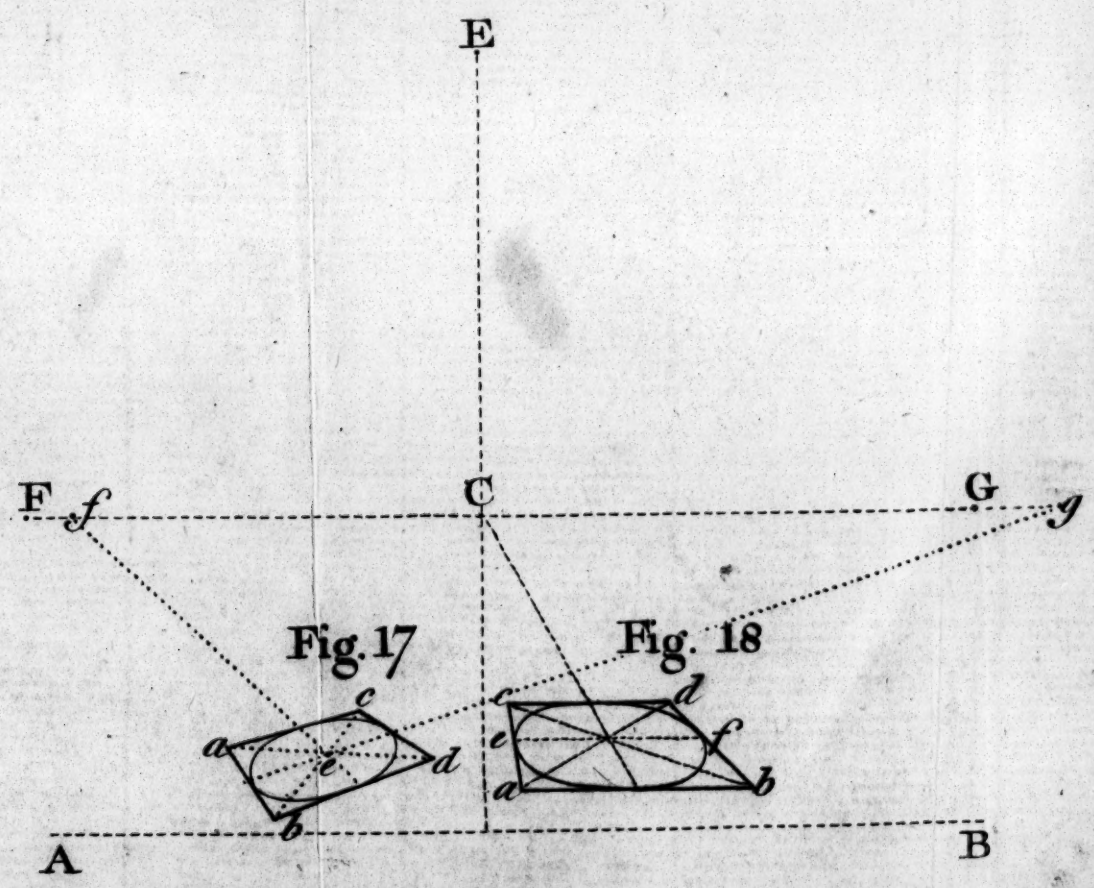
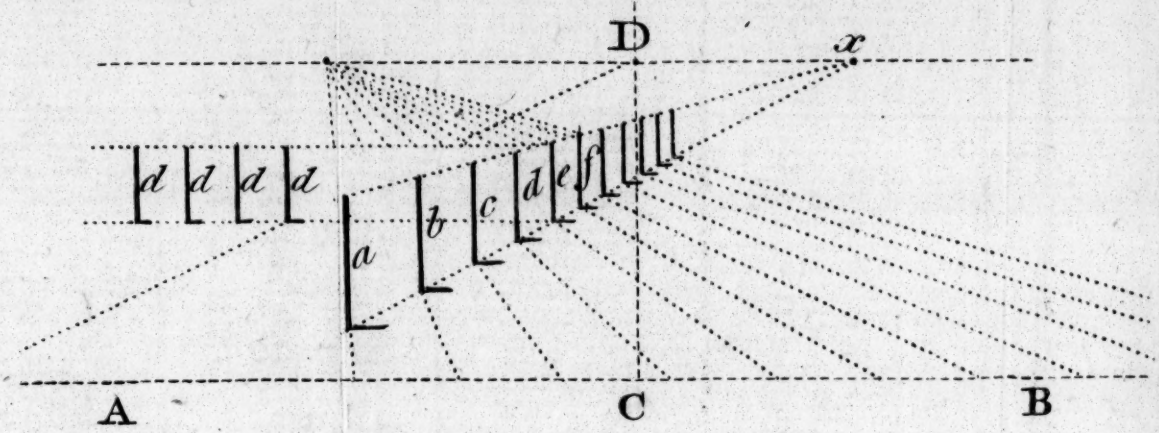


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Fig. 16



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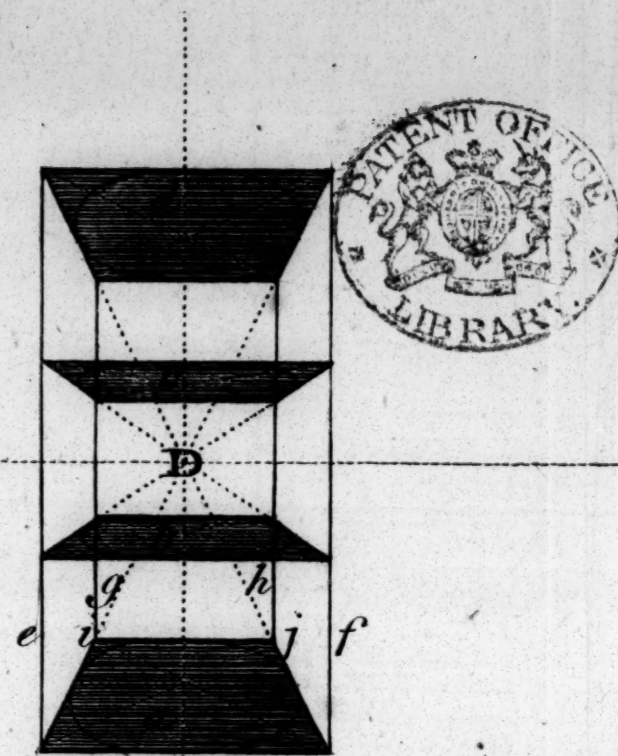
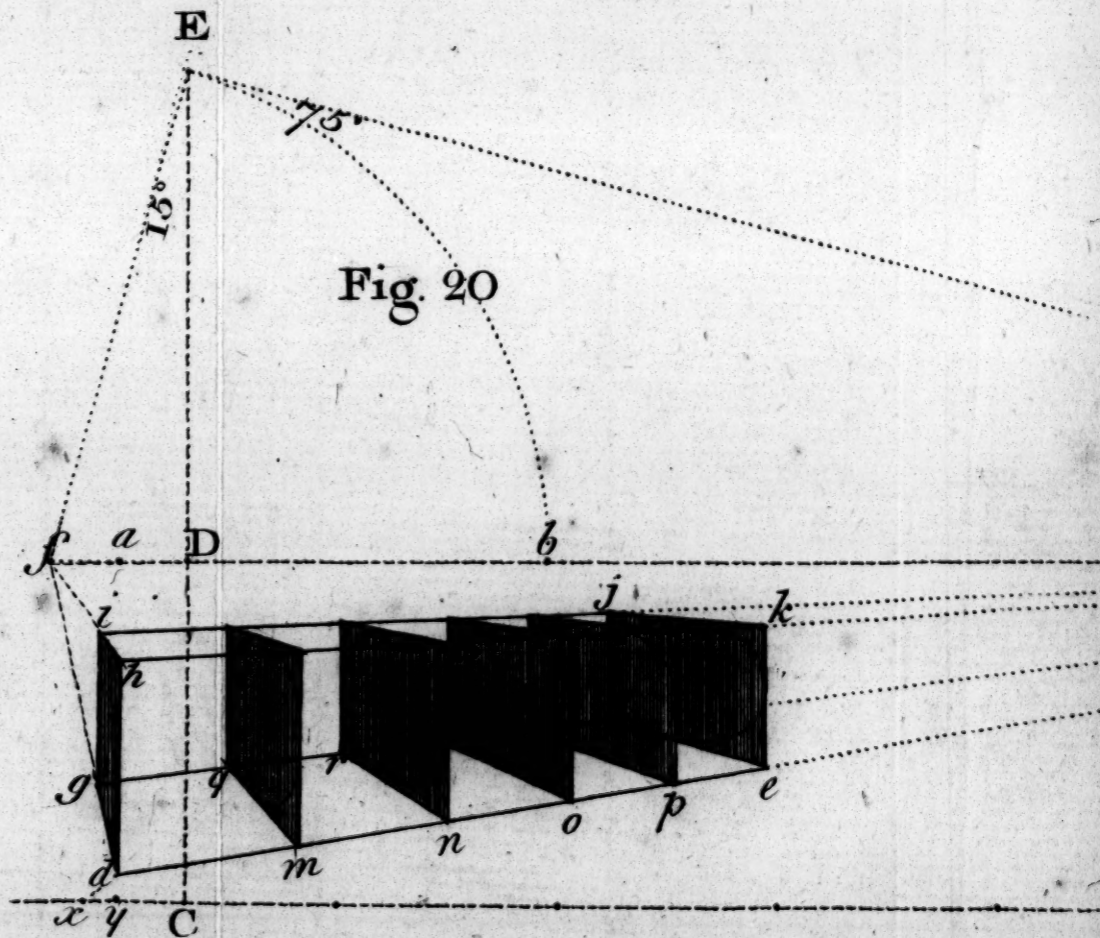


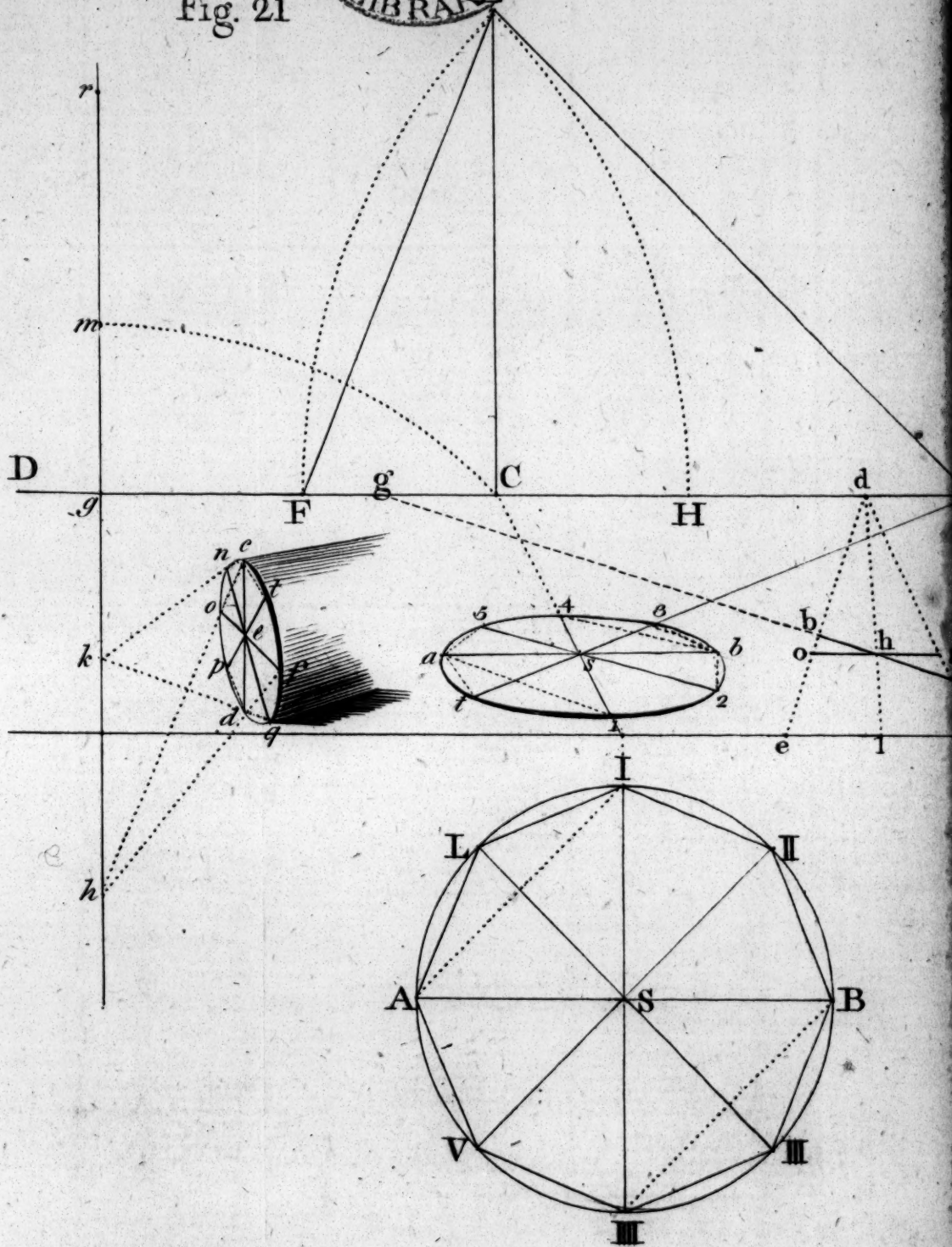
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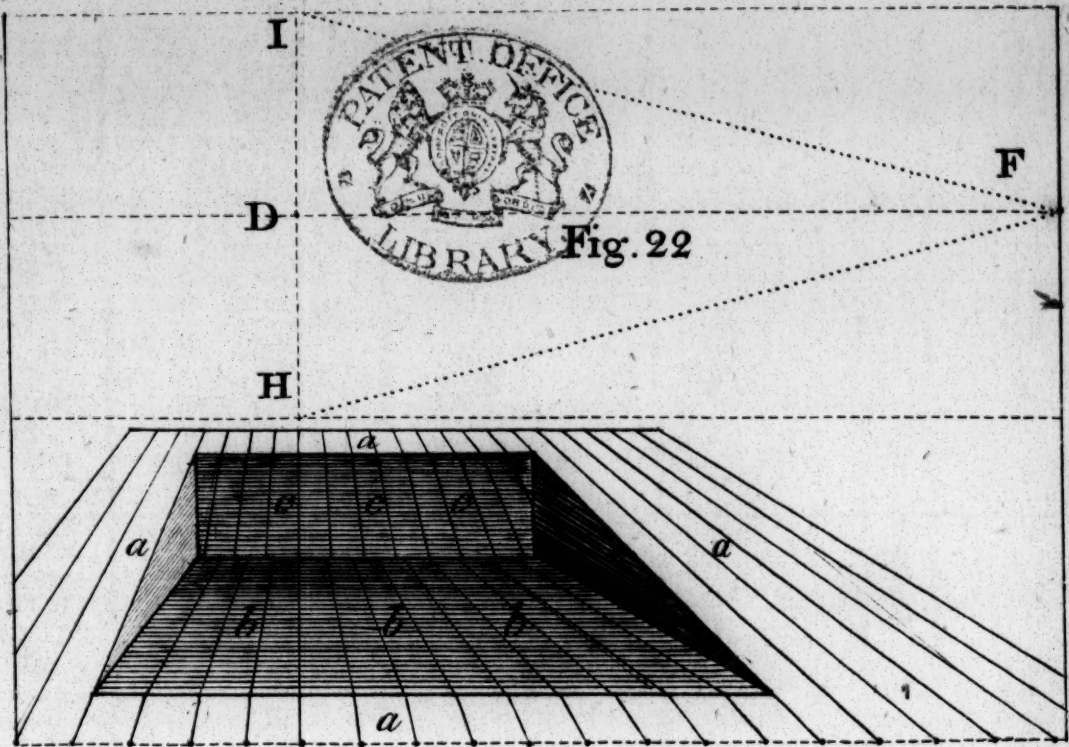
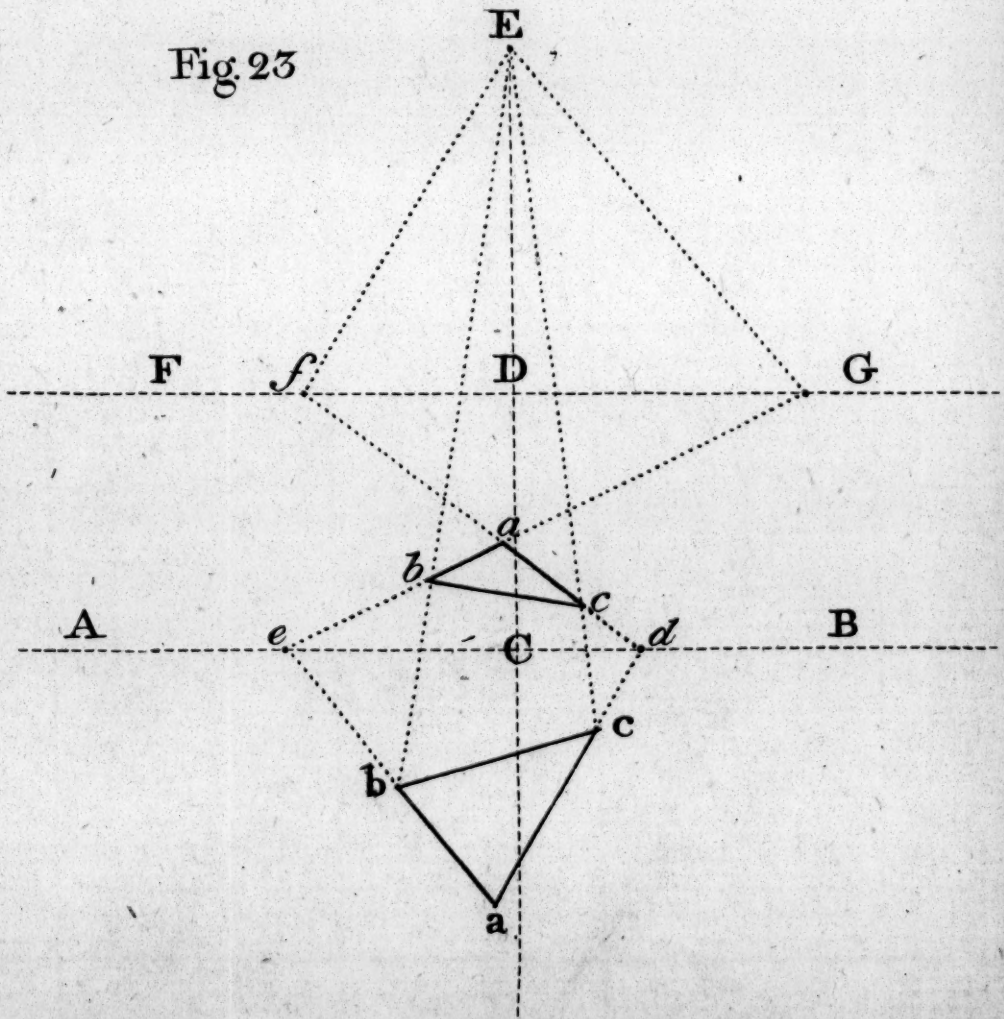


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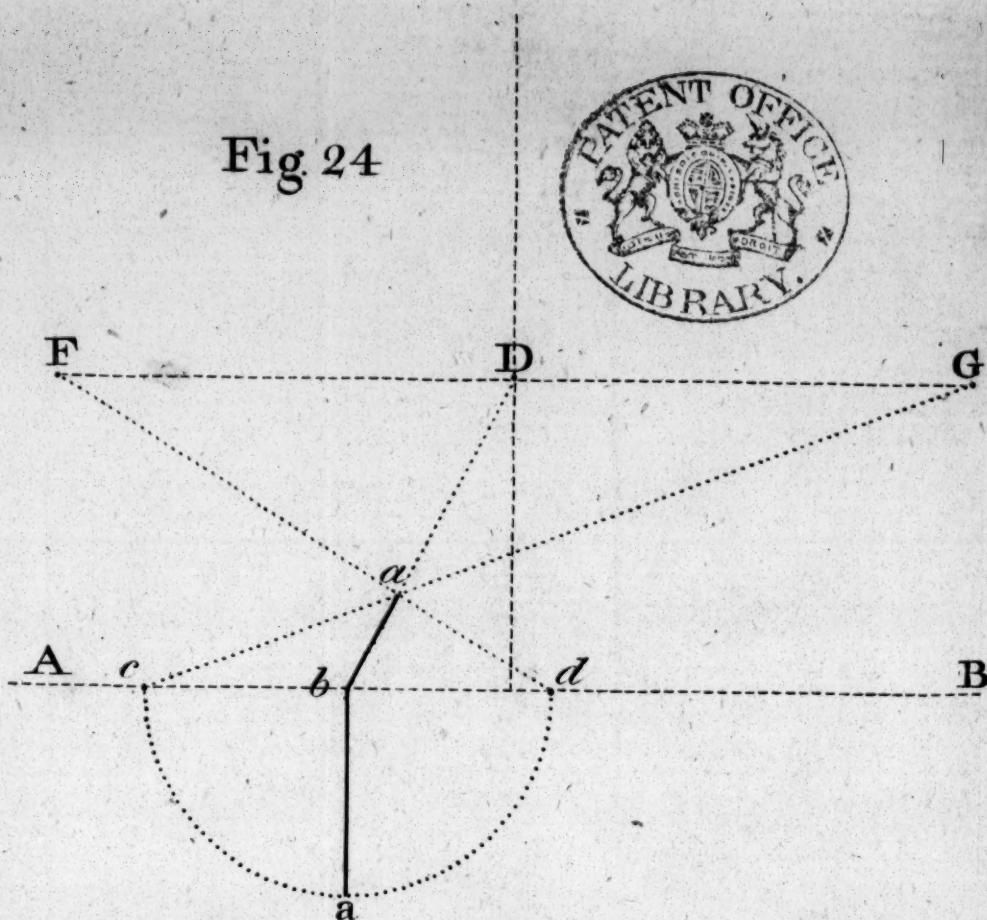
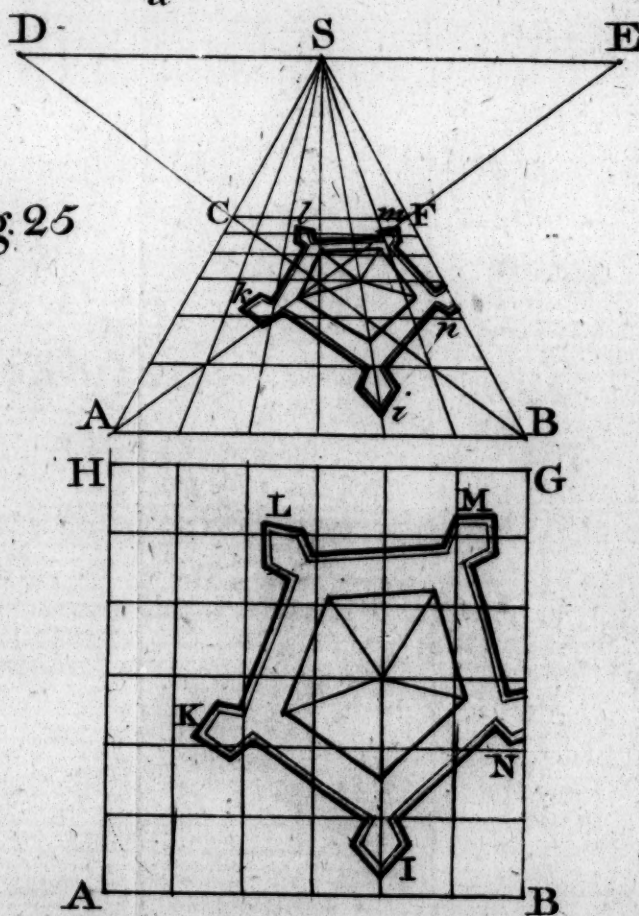


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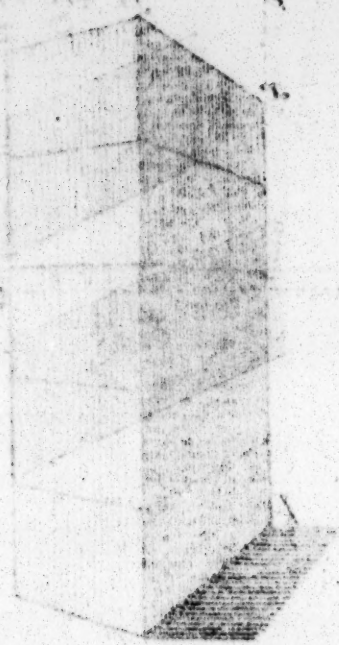


Fig. 20



Fig. 21



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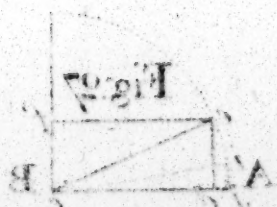


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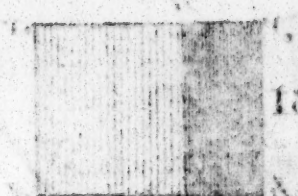


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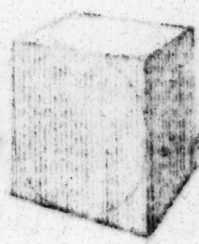


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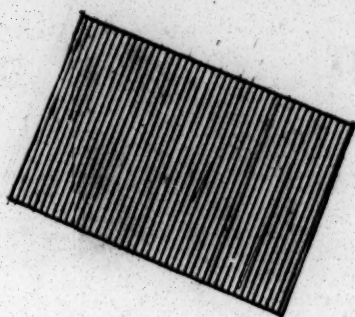


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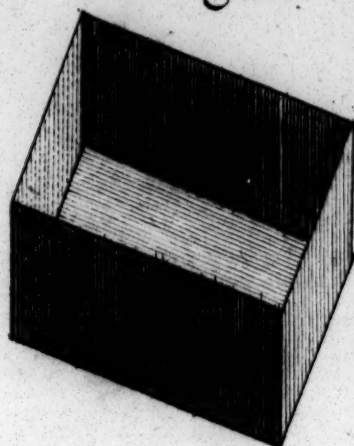
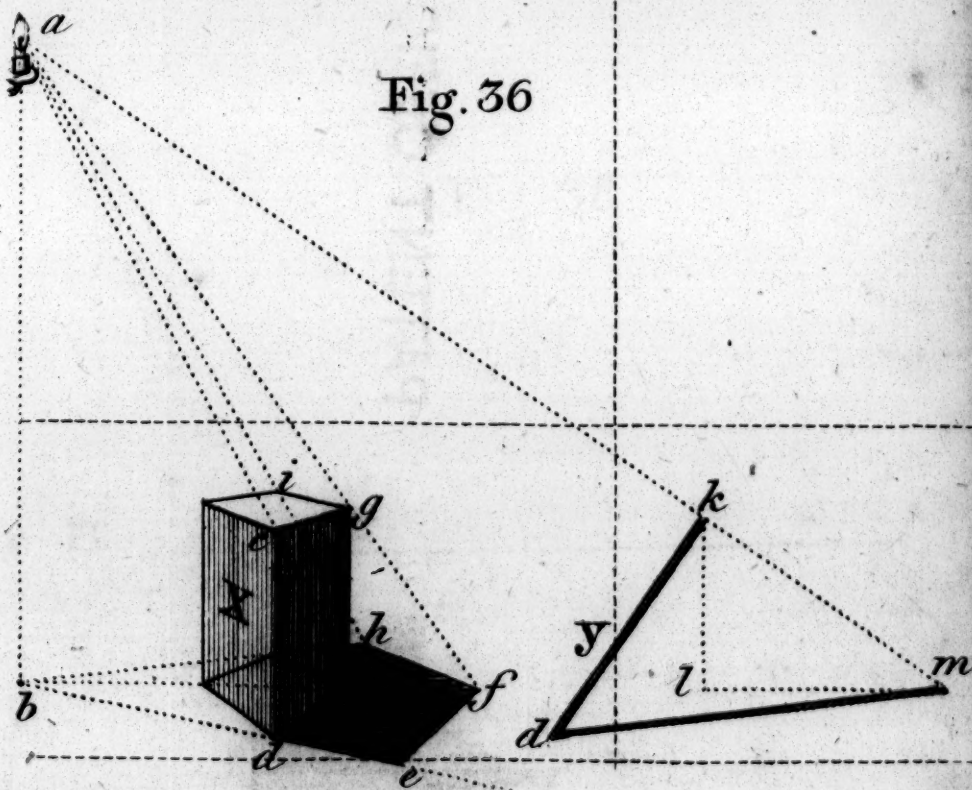
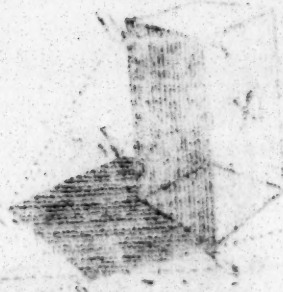
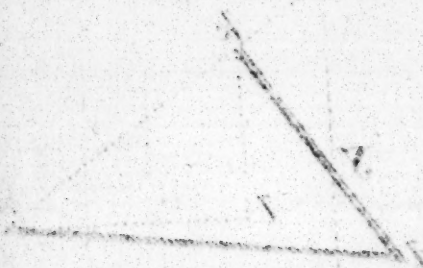
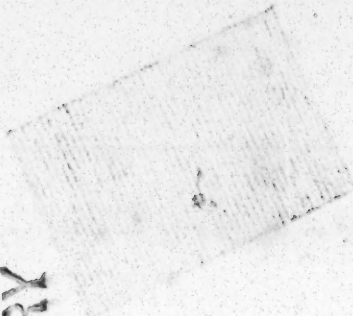


Fig. 36





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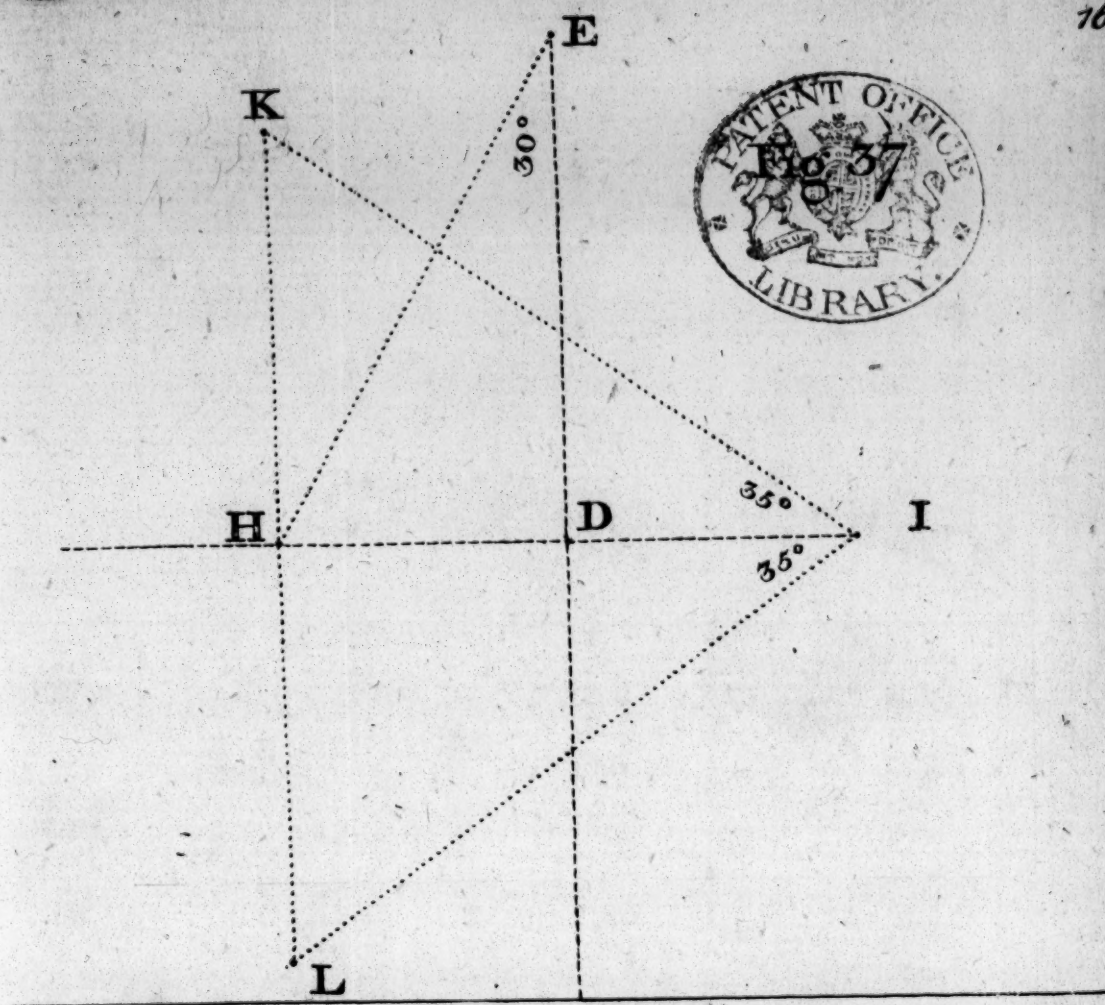
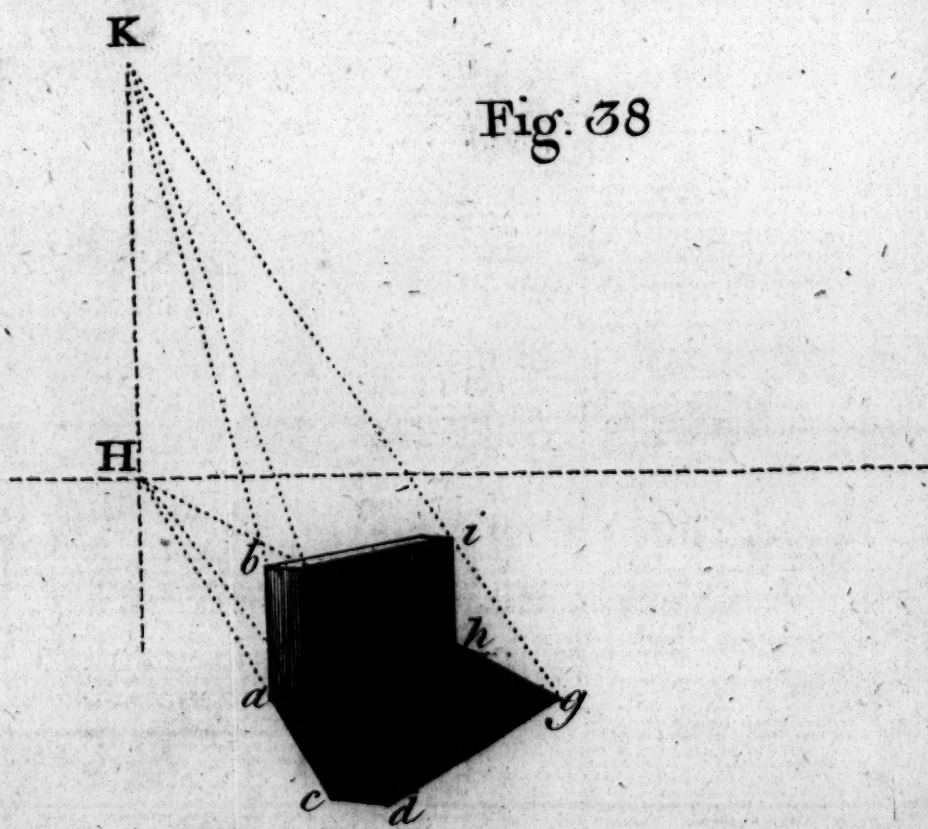


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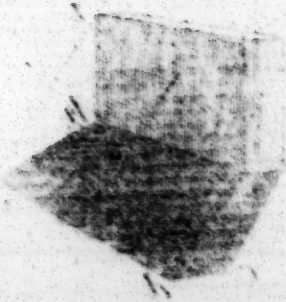


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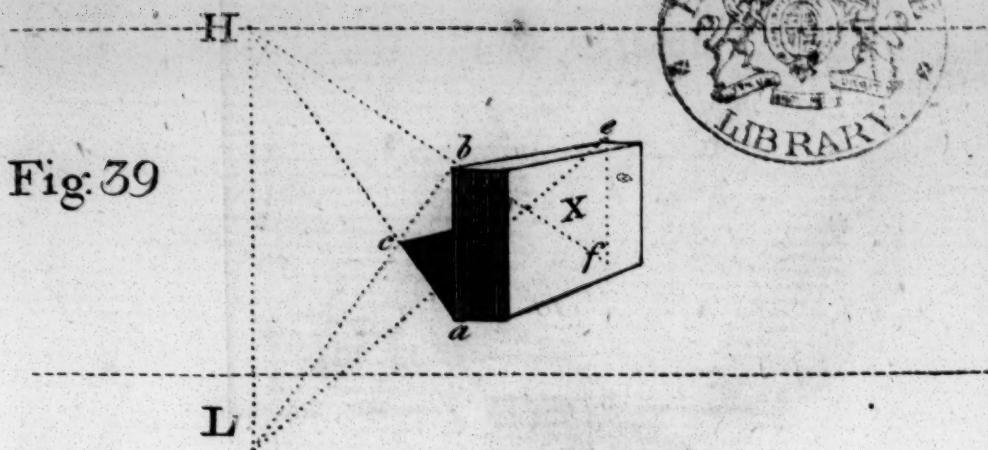


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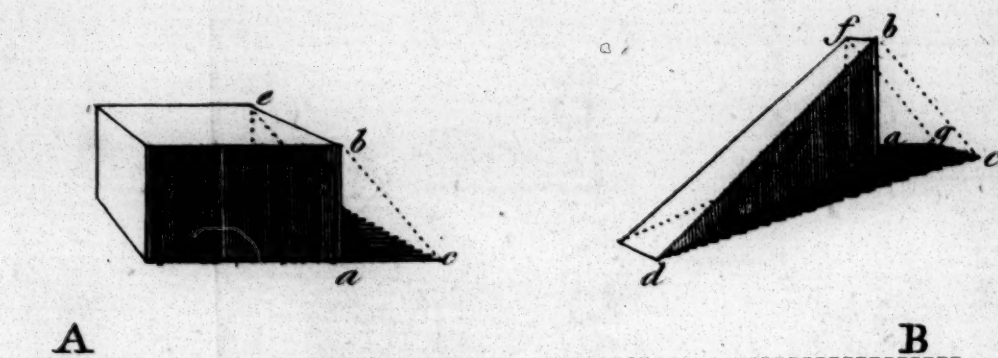
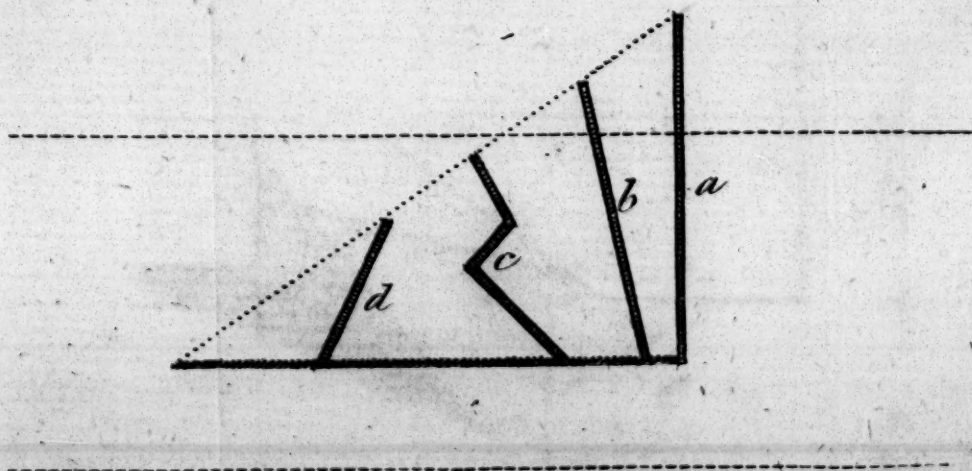


Fig. 41



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Fig. 42

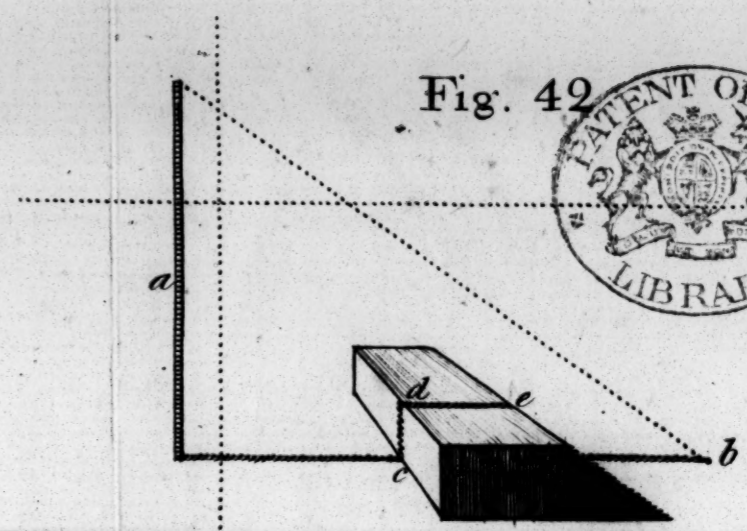


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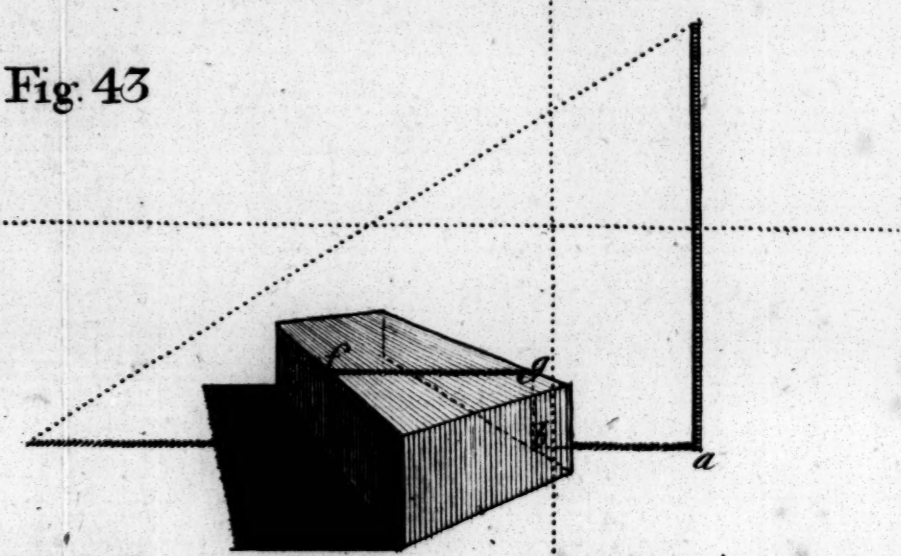
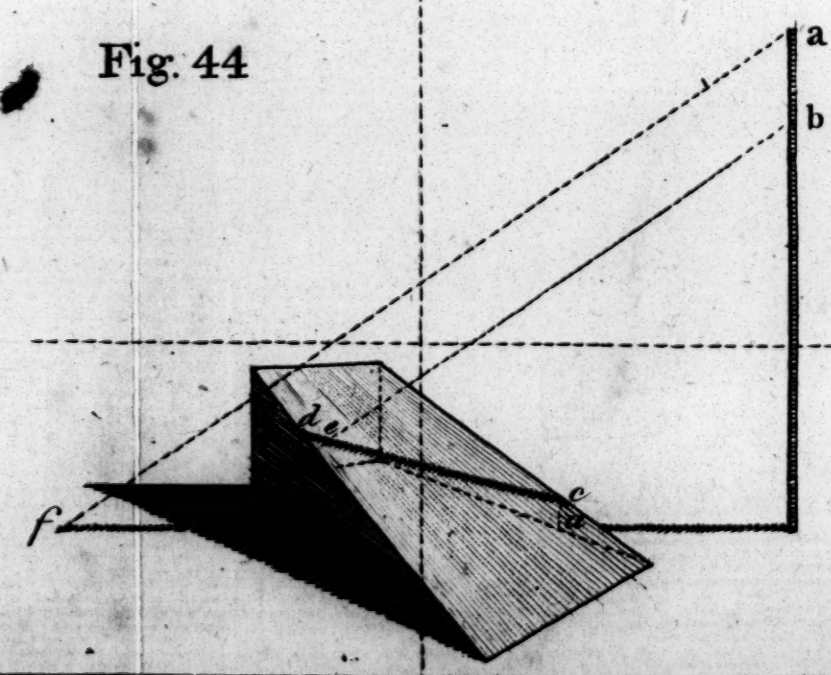


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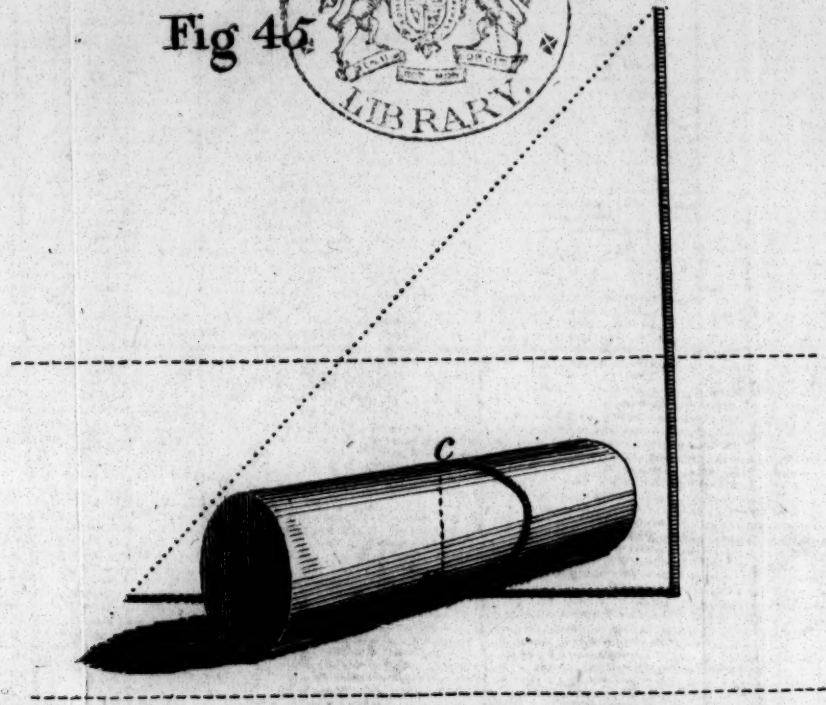
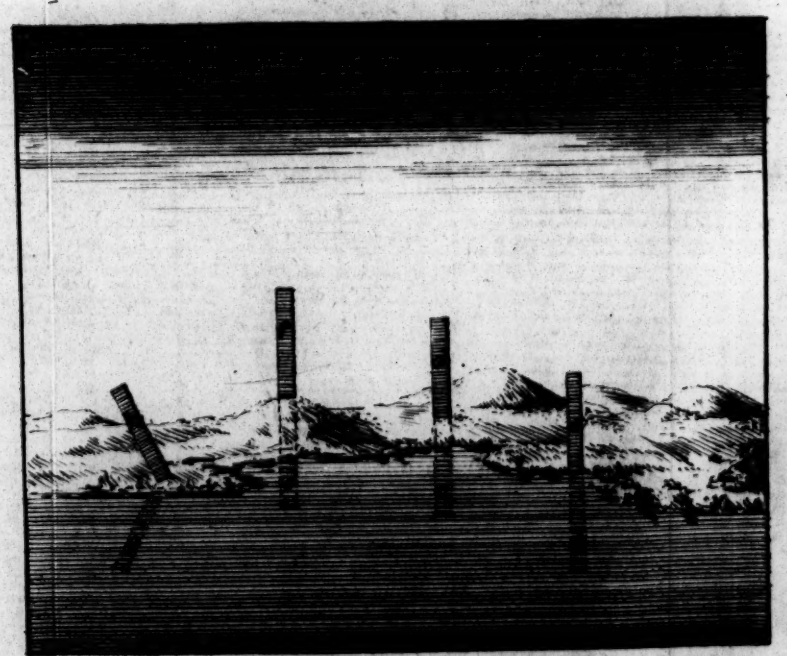


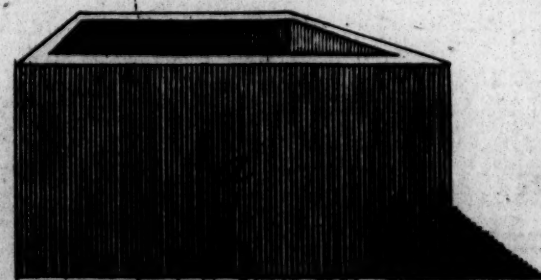
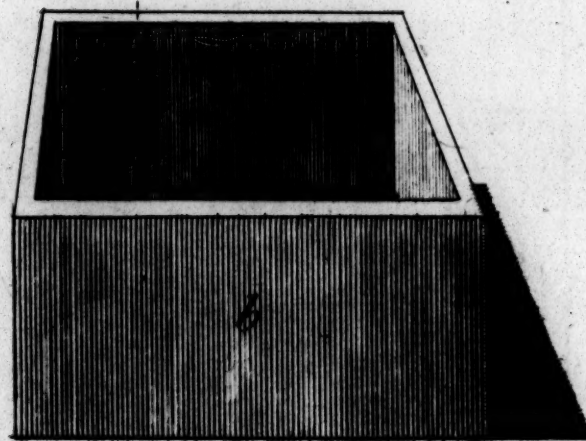
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Fig. 47



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Fig. 48

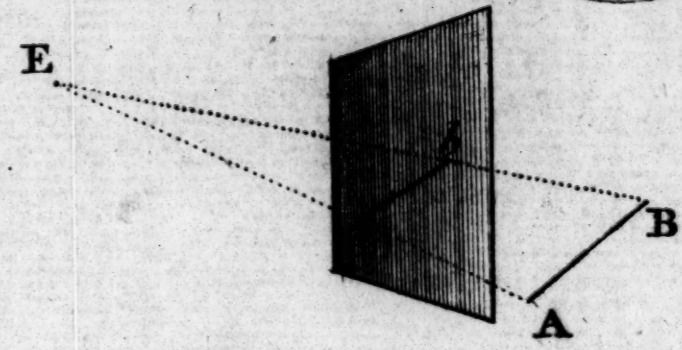
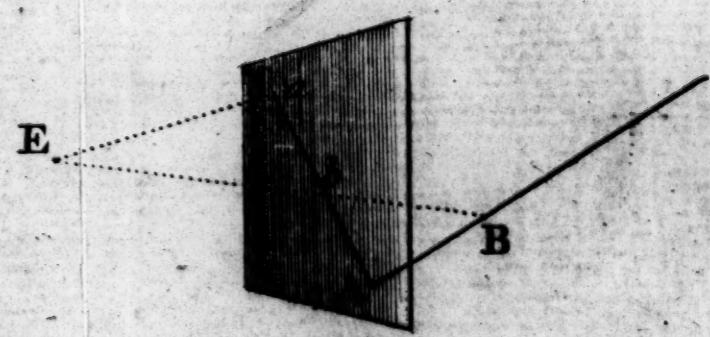


Fig. 49



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Fig. 50

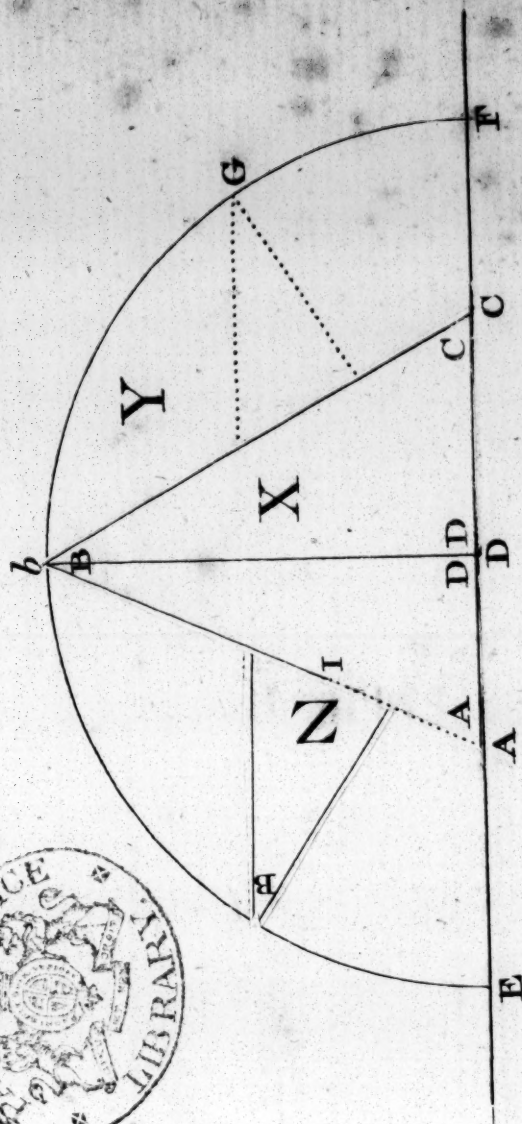
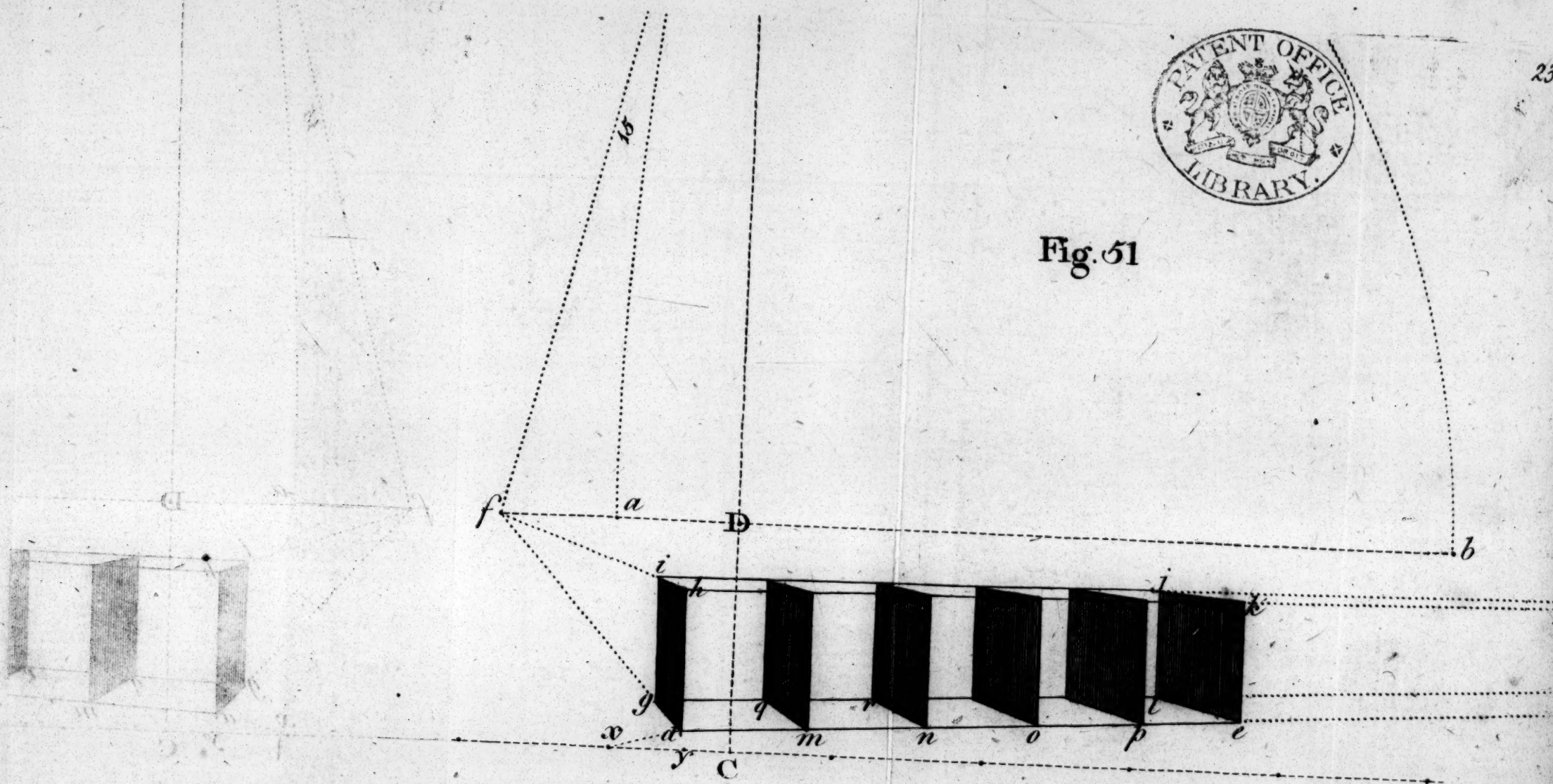




Fig. 51



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